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Generalization of Low Separation Axiom

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تعميم بديهيات الفصل الضعيفة

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Abstract:
In this article, researchers utilized the concepts of generalized closed sets, generalized regular closed sets and regular generalized closed sets to introduce new spaces for space $T_{\frac{3}{4}}$, namely generalized $T_{\frac{3}{4}}$, generalized regular $T_{\frac{3}{4}}$ and regular generalized $T_{\frac{3}{4}}$ spaces and they are represented by g-$T_{\frac{3}{4}}$ (gr-$T_{\frac{3}{4}}$, rg-$T_{\frac{3}{4}}$) spaces. The article investigates the relationship between them, as well as the relationship between g-$T_{\frac{3}{4}}$ space, T_1 space and $T_{\frac{3}{4}}$ space. In addition, researchers show that there is no relationship between g-$T_{\frac{3}{4}}$ space and $T_{\frac{1}{2}}$ space. Furthermore, researchers characterize the topological inheritance properties of these spaces. Finally, researchers studied the behavior of these spaces in regular space, $T_{\frac{1}{2}}$ space, partition space and extremely disconnected.

Keywords: Regular Open Sets, Generalized Closed Sets, Regular Generalized Closed Sets, Generalized Regular Closed Sets, $T_{\frac{3}{4}}$ Spaces.

الملخص
في هذه الورقة, نستخدم مفاهيم المجموعات المغلقة المعممة (المغلقة المعممة المنتظمة, المغلقة المنتظمة المعممة) لتقديم تعريفات جديدة للفضاء $T_{\frac{3}{4}}$ سميناها بالفضاءات $T_{\frac{3}{4}}$ المعممة, $T_{\frac{3}{4}}$ المعممة المنتظمة, $T_{\frac{3}{4}}$ المنتظمة المعممة ويتم التعبير عنا g-$T_{\frac{3}{4}}$ (gr-$T_{\frac{3}{4}}$, rg-$T_{\frac{3}{4}}$) وقمنا بدراسة العلاقة بين هذه الفضاءات ببعض, وعلاقة الفضاء $T_{\frac{3}{4}}$ المعمم بالفضاءات T_1 و $T_{\frac{3}{4}}$. بالإضافة إلى ذلك، نوضح أنه لا توجد علاقة بين الفضاء g-$T_{\frac{3}{4}}$ و الفضاء $T_{\frac{1}{2}}$. علاوة على ذلك قمنا بدراسة الخصائص الوراثية لهذه الفضاءات التبولوجية. وأخيرا تم دراسة سلوك هذه الفضاءات في الفضاء المنتظم و الفضاء $T_{\frac{1}{2}}$ والفضاء (Partition) والفضاء (Extremely Disconnected) .

الكلمات المفتاحية: المجموعات المفتوحة المنتظمة، المجموعات المغلقة المعممة، المجموعات المغلقة المعممة المنتظمة، المجموعات المغلقة المنتظمة المعممة، الفضاء $T_{\frac{3}{4}}$.

Introduction

The Stone [1] introduced and studied regular open sets. Reference [2] by Levine in 1970, Levine introduced the concept of a generalized closed set ($\mathfrak{g}$-closed) in topological space. In 1993 Palaniappan and Rao [3] introduced the concept of regular generalized closed sets (r$\mathfrak{g}$-closed) and he proved that this class of sets is weaker than the class of $\mathfrak{g}$-closed sets. In 2011, Bhattacharya [4] introduced a novel category of sets known as generalized regular closed sets ($\mathfrak{g}$r-closed), as part of his study on their behavior in relation to unions, intersection, and subspace. Maitra introduced the concept of $\mathfrak{g}$-closure and $\mathfrak{g}$-interior in 2015 [5]. In 1995 Balachandran, Sundaram and Maki [6] introduced the concept of generalized continuous maps, known as $\mathfrak{g}$-continuous maps, by utilizing $\mathfrak{g}$-closed sets and they explored some of their properties. In 1970, [2] utilized the idea of generalized closed sets to introduce a type of a space known $T_{\frac{1}{2}}$ space, demonstrating that properties such as compactness, locally compactness, countable compactness, Para-compactness and normality, among others, are all hereditary under $\mathfrak{g}$-closed conditions. Researcher also introduced a separation axiom named $T_{\frac{1}{2}}$, which lies between T_1 andT_0. Recently, several modifications have been identified and studied. These modifications were applied to provide a set of separate low axioms. In 1977 Dunham [7] demonstrated that a space is $T_{\frac{1}{2}}$ if and only if every singleton set is open or closed. In [8, 9] Arenas, Dontchev, Janster, Maki, Umehara and Yammura, presented a series of separation axioms between the spaces T_0 and T_1. In particular, they defined the spaces $T_{\frac{1}{4}}$, $T_{\frac{1}{3}}$, $T_{\frac{1}{2}}$ and $T_{\frac{3}{4}}$ using the concepts of ∧-sets, generalized ∧-sets, λ-sets, generalized closed sets and regular open sets to establish the separation axioms of these spaces [10, 11]. Additionally, Sarsak [12] defined the axioms μ-$T_{\frac{1}{4}}$, μ-$T_{\frac{3}{4}}$ and μ-$T_{\frac{1}{2}}$ using the concept of μ-open sets. In 2017 Gompa [13] described further separation axioms between T_0 and T_1spaces as properties of the space at a specific point. In 1990 Kar and Bhattacharyye [14] defined spaces that include the previous axioms, such as pre- T_0, pre- T_1, and pre- T_2, in addition to pre-regular and pre-normal spaces, These concept have been defined by many researchers by replacing the notion of open sets with pre-open sets in the classical definitions. In 1995 Dontchev [15] defined pre-$T_{\frac{1}{2}}$ space as a space in which every singleton set is either pre-open or pre-closed. He utilized the property stating that any singleton set is either pre-open or nowhere dense, which was proven by Jankovic and Reilly [16], to demonstrate that any topological space is pre-$T_{\frac{1}{2}}$ space. Since every topological space is pre-$T_{\frac{1}{2}}$, researchers seek to define a weaker space than the pre-T_1 space and the $T_{\frac{3}{4}}$-space, which is referred to as the pre-$T_{\frac{3}{4}}$ space. In 2022 Abdeen and Arwini [17] used the concept of regular open sets and pre-open sets study the topological properties of this space, and then examined its relationship with the classical separation axioms.

In this paper, researchers use the concepts of generalized closed sets, generalized regular closed and regular generalized closed sets to introduce new spaces for space $T_{\frac{3}{4}}$ namely $\mathfrak{g}$-$T_{\frac{3}{4}}$ space, $\mathfrak{g}$r-$T_{\frac{3}{4}}$ space and r$\mathfrak{g}$ -$T_{\frac{3}{4}}$ space. The space $\mathfrak{g}$r-$T_{\frac{3}{4}}$ is stronger than both $\mathfrak{g}$-$T_{\frac{3}{4}}$ space and r $\mathfrak{g}$-$T_{\frac{3}{4}}$ space. Researchers examine the relationship between these two as well as the relationship between $\mathfrak{g}$-$T_{\frac{3}{4}}$ space and T_1 space and $T_{\frac{3}{4}}$. In addition, researchers show that there is no relationship between $\mathfrak{g}$-$T_{\frac{3}{4}}$ space and $T_{\frac{1}{2}}$ space. Furthermore, researchers characterize the topological inheritance properties of these spaces. Finally, researchers invistigated the behavior of these spaces in regular space, $T_{\frac{1}{2}}$ space, partition space and extremely disconnected.

1. Preliminaries

Definition 1.1. [1,18] If B = $\overset{\circ}{\overline{B}}$, then the set B in a topological space Z is defined as regular open set (r-open). The complement of a regular open set is called a regular closed set (r-closed). The family of all r-closed sets in a space Z is denoted by RC(Z) and the family of all r-open sets in a space is denoted by RO(Z).

Definition 1.2. [2] if $\overline{B}$ ⊆V, when B⊆V and V is an open set, then a subset B of a topological space Z is defined as generalized closed set ($\mathfrak{g}$-closed). The complement of a $\mathfrak{g}$-closed set is called $\mathfrak{g}$-open set. The family of all $\mathfrak{g}$-closed sets in a space Z is denoted by $\mathfrak{g}$C(Z) and the family of all $\mathfrak{g}$-open sets in a space is denoted by $\mathfrak{g}$O(Z). Z

Definition 1.3. [3] A subset B of a topological space Z is said to be regular generalized closed set (r g-closed) if $\overline{B}$⊆V whenever B⊆V and V is regular open in Z.

Definition 1.4. [4] A subset B of a topological space Z is called generalized regular closed set (gr-closed) if $\overline{B}^{r}$⊆V whenever B⊆V and V is open in Z**.**

Corollary 1.1. [4] In any topological space Z, the following statements are considered true:
1. Any r-closed set is gr-closed.
2. Any gr-closed set is g-closed.
3. Any g-closed set is r g-closed.

Theorem 1.1. [19,11,3] A function F: Z ⟶ Y is defined as follows:
1. Strongly regular open map if the image of each r-open subset of Z is r-open in Y.
2. Continuous and closed map if the image of each g-closed subset of Z is g-closed in Y.
3. Regular irresolute and closed map if the image of each rg-closed subset of Z is rg-cloaed in Y.

Definition 1.5. [19] A space Z is called a regular space if for any element z ∊ Z, and for any open set V∊ Z, s.t. z ∊ V there is an open set U such that z ∊ U ⊆ $\overline{U}$ ⊆ V**.**

Corollary 1.2. [20]
1. In the regular space Z, any subset B is g-closed iff B is gr-closed.
2. In the regular space Z, if B⊆Z, then $\overline{B}^{r}=\overline{B}$.

Definition 1.6. [17] If Z is a topological space, and any open set of Z is closed set, then Z said to be a " partition space"

Corollary 1.3. [4] Any subset of partition space Z is r g-closed.

Definition 1.7. [20] The topological space Z is called "extremely disconnected" If the closure of any open set of Z is open.

Corollary 1.4. [20] In the extremely disconnected space Z any subset is rg-closed**.**

Definition 1.8. [17] A topological space Z is considered a " T_1 Space" if all singletons are closed.

Definition 1.9. [21] The topological space Z is considered a "$T_{\frac{3}{4}}$ Space" if all singletons are either closed or r-open.

Definition 1.10. [17] The topological space Z is considered a "$T_{\frac{1}{2}}$ Space" if all singletons are either closed or open.

Corollry 1.5. [21]
1. In $T_{\frac{1}{2}}$ space any g-closed set is closed.
2. Any T_1-space is $T_{\frac{3}{4}}$.
3. Any $T_{\frac{3}{4}}$-space is $T_{\frac{1}{2}}$.

GENERALIZED $T_{\frac{3}{4}}$ SPACES

Definition 2.1.
A topological space Z is said to be an generalized $T_{\frac{3}{4}}$ space(g-$T_{\frac{3}{4}}$), if any singleton {z}, where z ∊Z is either an r-open or g-closed.

Proposition 2.1.
1. Any T_1 space is g-$T_{\frac{3}{4}}$.
2. Any $T_{\frac{3}{4}}$ space is g-$T_{\frac{3}{4}}$.

Proof:

1. Assume that Z is T_1-space, thus {z} is closed. Becouse any closed set is $\mathfrak{g}$-closed, {z} is either r-open or $\mathfrak{g}$-closed, which means that Z is a $\mathfrak{g}$-$T_{\frac{3}{4}}$ space.

2. This is evident, as any closed set is g-closed.

$$T_1 \longrightarrow \mathfrak{g}\text{-}T_{\frac{3}{4}}$$
$$T_{\frac{3}{4}} \longrightarrow \mathfrak{g}\text{-}T_{\frac{3}{4}}$$

Corollary 2.1.
There is no relationship between a $T_{\frac{1}{2}}$ space and a $\mathfrak{g}$-$T_{\frac{3}{4}}$.

$$T_{\frac{1}{2}} \nrightarrow \mathfrak{g}\text{-}T_{\frac{3}{4}}$$
$$\mathfrak{g}\text{-}T_{\frac{3}{4}} \nrightarrow T_{\frac{1}{2}}$$

Examples 2.1.

1. If Z = {m, n, r}, τ= {Z, ∅, {m}, {r}, {m,r}} thus Z is $T_{\frac{1}{2}}$ and $\mathfrak{g}$-$T_{\frac{3}{4}}$ but it is not T_1 .
2. If Z = {1, 2, 3 }, τ= {Z, ∅, {1}, {2, 3}} then Z is $\mathfrak{g}$-$T_{\frac{3}{4}}$ but it is neither $T_{\frac{1}{2}}$ nor $T_{\frac{3}{4}}$.
3. 1. If Z = {a, b}, τ= {Z, ∅, {a}}, thus Z is $T_{\frac{1}{2}}$ and not $\mathfrak{g}$-$T_{\frac{3}{4}}$.

Corollary 2.2.
If Z is $\mathfrak{g}$-$T_{\frac{3}{4}}$ space, then {z} is open or $\mathfrak{g}$-closed.

Proof:
By definition (2.1) and becouse any r-open set is an open set.

Proposition 2.2.
Any $T_{\frac{1}{2}}$ regular space is $\mathfrak{g}$-$T_{\frac{3}{4}}$.

Proof:
Let Z be $T_{\frac{1}{2}}$ space, thus {z} is an open or closed, and since the open set in regular space is r-open. Then {z} is an r-open or closed (because any closed is $\mathfrak{g}$- closed). Therefore Z is $\mathfrak{g}$-$T_{\frac{3}{4}}$.

$$T_{\frac{1}{2}} \xrightarrow{\text{regular space}} \mathfrak{g}\text{-}T_{\frac{3}{4}}$$

Theorem 2.1.
In $T_{\frac{1}{2}}$ space the following conditions are equivalent:

1. The space $\mathfrak{g}$-$T_{\frac{3}{4}}$ is $T_{\frac{3}{4}}$.
2. The space $\mathfrak{g}$-$T_{\frac{3}{4}}$ is T_1.

Proof:
1. Directly from corollary(1.5)
2. Directly from corollary(1.5)

$$T_{\frac{3}{4}} \overset{T_{\frac{1}{2}}\ space}{\Longleftrightarrow} \mathfrak{g}\text{-}T_{\frac{3}{4}}$$
$$T_1 \overset{T_{\frac{1}{2}}\ space}{\Longleftrightarrow} \mathfrak{g}\text{-}T_{\frac{3}{4}}$$

Theorem 2.2. The image of a $\mathfrak{g}$-$T_{\frac{3}{4}}$ space under a continuous, closed and strongly r-open map is a $\mathfrak{g}$-$T_{\frac{3}{4}}$ space.

Proof: Assume that φ:Z→W be a comprehensive function from the $\mathfrak{g}$-$T_{\frac{3}{4}}$ space Z, For any element w

∈ W, there is a point z∈Z s.t. φ (z)=w. Because Z is $\mathfrak{g}$-$T_{\frac{3}{4}}$ space, the set {z} is either r-open or $\mathfrak{g}$-closed in Z. Given that the functions φ is continuous, closed and strongly r-open, F(z)={w} will be either r-open or $\mathfrak{g}$-closed in W.

Definition 3.1.
A topological space Z is said to be regular generalized $T_{\frac{3}{4}}$ space (r$\mathfrak{g}$-$T_{\frac{3}{4}}$). If any singleton {z}, where z ∈ Z is either an r-open or r$\mathfrak{g}$-closed.

Proposition 3.1.
1. Any $\mathfrak{g}$-$T_{\frac{3}{4}}$ space is r$\mathfrak{g}$-$T_{\frac{3}{4}}$.
2. Any $T_{\frac{3}{4}}$ space is r$\mathfrak{g}$-$T_{\frac{3}{4}}$.

Proof:
1. Direct since any $\mathfrak{g}$-closed set of Z is r$\mathfrak{g}$- closed.
2. Direct since any closed set is $\mathfrak{g}$-closed and $\mathfrak{g}$-closed is r$\mathfrak{g}$- closed.

$$T_{\frac{3}{4}} \longrightarrow \mathfrak{g}\text{-}T_{\frac{3}{4}} \longrightarrow r\mathfrak{g}\text{-}T_{\frac{3}{4}}$$

Example 3.1.
If Z = {m, n, r }, τ= {Z, ∅, {m,n}, {n,r}, {n}} then Z is an r$\mathfrak{g}$-$T_{\frac{3}{4}}$ space but is nether $\mathfrak{g}$-$T_{\frac{3}{4}}$ nor $T_{\frac{3}{4}}$.

Propostion 3.2.
In regular space $\mathfrak{g}$-$T_{\frac{3}{4}}$ space and r$\mathfrak{g}$-$T_{\frac{3}{4}}$ are equivalent.

Proof:
Let B⊆V, where V is open since Z is regular space. Therefore V is r-open. However, because B is r$\mathfrak{g}$-closed, researchers have $\overline{B}$ ⊆V. Thus B is $\mathfrak{g}$-closed.

$$r\mathfrak{g}\text{-}T_{\frac{3}{4}} \xrightarrow{\text{regular space}} \mathfrak{g}\text{-}T_{\frac{3}{4}}$$

Proposition 3.3.
In regular $T_{\frac{1}{2}}$ space, any r$\mathfrak{g}$-$T_{\frac{3}{4}}$ space is $T_{\frac{3}{4}}$.

Proof:
Direct since in regular space any r$\mathfrak{g}$-closed set is $\mathfrak{g}$-closed, since in $T_{\frac{1}{2}}$ space any $\mathfrak{g}$-closed set is closed, then the space is $T_{\frac{3}{4}}$.

$$r\mathfrak{g}\text{-}T_{\frac{3}{4}} \xrightarrow{T_{\frac{1}{2}}\ \text{regular space}} T_{\frac{3}{4}}$$

Theorem 3.1.
The image of an r $\mathfrak{g}$-$T_{\frac{3}{4}}$ space under a r-irresolute, closed and strongly r-open map is r$\mathfrak{g}$-$T_{\frac{3}{4}}$ space.

Proof:
Assume that φ:Z→W be a comprehensive mapping from the r$\mathfrak{g}$-$T_{\frac{3}{4}}$ space Z, For any element w ∈ W, there is a point z∈Z s.t. φ (z)=w. Because Z is r$\mathfrak{g}$-$T_{\frac{3}{4}}$ space, the set {z} is either r-open or r$\mathfrak{g}$-closed in Z. Given that the φ is r--irresolute, closed and strongly r-open, φ(z)={w} will be either r-open or r$\mathfrak{g}$-closed in W.

GENERALIZED REGULAR $T_{\frac{3}{4}}$ SPACES

Definition 4.1.
A topological space Z is said to be generalized regular space $T_{\frac{3}{4}}$ ($\mathfrak{g}$r-$T_{\frac{3}{4}}$), if any singleton {z}, where z ∈ Z, is either an r-open or $\mathfrak{g}$r-closed.

Proposition 4.1.

1. Any $\mathfrak{gr}$-$T_{\frac{3}{4}}$ is $\mathfrak{g}$-$T_{\frac{3}{4}}$.
2. Any $\mathfrak{gr}$-$T_{\frac{3}{4}}$ is $r\mathfrak{g}$-$T_{\frac{3}{4}}$.

Proof:

1. Let Z be an $\mathfrak{gr}$-$T_{\frac{3}{4}}$ space, i.e for each z ϵ Z, {z} is r-open or $\mathfrak{gr}$-closed since an $\mathfrak{gr}$-closed in Z is $\mathfrak{g}$-closed, thus {z} is r-open or $\mathfrak{g}$-closed. Therefore Z is $\mathfrak{g}$-$T_{\frac{3}{4}}$

2. Let Z be an $\mathfrak{gr}$-$T_{\frac{3}{4}}$ space, i.e {z} is r-open or $\mathfrak{gr}$- closed sice any $\mathfrak{gr}$-closed is $r\mathfrak{g}$-closed. Then Z is $r\mathfrak{g}$-$T_{\frac{3}{4}}$.

$$\mathfrak{gr}\text{-}T_{\frac{3}{4}} \longrightarrow \mathfrak{g}\text{-}T_{\frac{3}{4}}$$

$$\mathfrak{gr}\text{-}T_{\frac{3}{4}} \longrightarrow r\mathfrak{g}\text{-}T_{\frac{3}{4}}$$

Corollary 4.1.

If Z a topological space, and RO (Z) = {Z, ∅}, then any subset of Z is $r\mathfrak{g}$-closed.

Examples 4.1.

1. If Z = {a, b, c }, τ= {Z, ∅, {a}, {b,c}}, RO {Z, ∅, {a}, {b,c}} then Z is $\mathfrak{gr}$-$T_{\frac{3}{4}}$ space.
2. Let Z = {a, b, c}, τ= {Z, ∅, {a, b}, {b, c}, {b}}, $\mathcal{F}$= {Z, ∅, {c}, {a}, {a, c}}, so RO=RC= {Z, ∅}, then Z is $r\mathfrak{g}$-$T_{\frac{3}{4}}$ but not $\mathfrak{g}$-$T_{\frac{3}{4}}$ nor $\mathfrak{gr}$-$T_{\frac{3}{4}}$.
3. The cofinite topology (IR,τ_C) is $\mathfrak{g}$-$T_{\frac{3}{4}}$, RO= {IR, ∅}. If B is any finite set there is open set V, but $\overline{B}^{r}$ = IR ⊈V. Hence (IR, τ_C) is not $\mathfrak{gr}$-$T_{\frac{3}{4}}$.

Proposition 4.2

In regular space $r\mathfrak{g}$-$T_{\frac{3}{4}}$ is $\mathfrak{gr}$-$T_{\frac{3}{4}}$.

Proof:

Direct since $r\mathfrak{g}$-$T_{\frac{3}{4}}$ regular is $\mathfrak{g}$-$T_{\frac{3}{4}}$. And $\mathfrak{g}$-$T_{\frac{3}{4}}$ regular is $\mathfrak{gr}$-$T_{\frac{3}{4}}$. Hance $r\mathfrak{g}$-$T_{\frac{3}{4}}$ regular $\mathfrak{gr}$-$T_{\frac{3}{4}}$.

$$r\mathfrak{g}\text{-}T_{\frac{3}{4}} \xrightarrow{\text{regular space}} \mathfrak{gr}\text{-}T_{\frac{3}{4}}$$

Theorem 4.1.

Any extremely disconnected space is $r\mathfrak{g}$-$T_{\frac{3}{4}}$.

Proof:

Direct since any subset of space Z is $r\mathfrak{g}$-closed. Then Z is $r\mathfrak{g}$-$T_{\frac{3}{4}}$.

$$\text{Extremely disconnected space} \longrightarrow r\mathfrak{g}\text{-}T_{\frac{3}{4}}$$

Theorem 4.2.

Any partition space is $\mathfrak{gr}$-$T_{\frac{3}{4}}$.

Proof:

Let Z be partition space and let z ϵ Z, any subset of Z is $\mathfrak{gr}$-closed then Z is $\mathfrak{gr}$-$T_{\frac{3}{4}}$ space.

$$\text{Partition space} \longrightarrow \mathfrak{gr}\text{-}T_{\frac{3}{4}}$$

Corollary 4.2.

Any partition space is $\mathfrak{g}$-$T_{\frac{3}{4}}$ ($r\mathfrak{g}$-$T_{\frac{3}{4}}$).

Proof:

Direct from "any $\mathfrak{gr}$-closed is $\mathfrak{g}$-closed ($r\mathfrak{g}$-closed)".

Conclusion

Using the concept of generalized closed sets, generalized regular closed and regular generalized closed sets. Researchers instigated a new space for the space $T_{\frac{3}{4}}$ namely $\mathfrak{g}$-$T_{\frac{3}{4}}$ space, $\mathfrak{gr}$-$T_{\frac{3}{4}}$ space and

rg-$T_{\frac{3}{4}}$ space. The space gr-$T_{\frac{3}{4}}$ is stronger than both g-$T_{\frac{3}{4}}$ space and rg-$T_{\frac{3}{4}}$ space. In this article, researchers address the relationship between theses with each other, and the relationship of g-$T_{\frac{3}{4}}$ space with the T_1 space and $T_{\frac{3}{4}}$. In addition, researchers have illustrated that there is no relationship between the g-$T_{\frac{3}{4}}$ and $T_{\frac{1}{2}}$ space. To conclude, researchers investigate the behavior of gr-$T_{\frac{3}{4}}$ (g-$T_{\frac{3}{4}}$, rg-$T_{\frac{3}{4}}$) space in some special. Here are some of our findings:

Any T_1 space is g-$T_{\frac{3}{4}}$.

Any $T_{\frac{3}{4}}$ space is g- $T_{\frac{3}{4}}$.

Any gr-$T_{\frac{3}{4}}$ space is g- $T_{\frac{3}{4}}$.

Any g-$T_{\frac{3}{4}}$ space is rg- $T_{\frac{3}{4}}$.

Any extremelly disconnected space is rg- $T_{\frac{3}{4}}$.

Any partition space is gr- $T_{\frac{3}{4}}$ (g-$T_{\frac{3}{4}}$, rg-$T_{\frac{3}{4}}$)

In $T_{\frac{1}{2}}$ space, these statments are whole:

Any g-$T_{\frac{3}{4}}$ space is $T_{\frac{3}{4}}$.

Any g-$T_{\frac{3}{4}}$ space is T_1.

In regular space, these statments are whole:

Any $T_{\frac{1}{2}}$ space is g-$T_{\frac{3}{4}}$.

Any rg-$T_{\frac{3}{4}}$ space is g- $T_{\frac{3}{4}}$.

Any g-$T_{\frac{3}{4}}$ space is gr- $T_{\frac{3}{4}}$.

In Regular $T_{\frac{1}{2}}$ space, every rg-$T_{\frac{3}{4}}$ space is $T_{\frac{3}{4}}$.

Regular-Irresolute closed and strongly r-open map preserves rg-$T_{\frac{3}{4}}$ spaces.

Continuous closed and strongly r-open map preserves g-$T_{\frac{3}{4}}$ spaces.

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