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Solitary Wave Solution for the Time-Fractional Modified KdV-Burger's Equation by Using the Residual Power Series Method

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الحل الموجي المنفرد لمعادلة كورتيج دي فريس و برجرز المعدلة الكسرية الزمنية باستخدام طريقة متسلسلة القوى المتبقية

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Abstract:
This study examines the usage of the residual power series method (RPSM) to numerically investigate the solution of the time-fractional modified KdV-Burger's equation in the Caputo's sense. The reliability and the efficiency of the method were demonstrated using 2D and 3D illustrations as well as error. The obtained results have indicated the potency of the RPSM for such initial values problems.

Keywords: Residual power series method, fractional modified KdV-Burger's equation, Caputo's sense

الملخص
هذه الدراسة تفحص استخدام طريقة متسلسلة القوى المتبقية (RPSM)، لتتحرى عددياً حل معادلة كورتيج دي فريس وبرجرز المعدلة الكسرية الزمنية بمعنى كابوتو. تم اثبات موثوقية وكفاءة الطريقة باستخدام رسومات توضيحية في بعدين وثلاثة ابعاد وكذلك الخطأ، اشارت النتائج التي تم الحصول عليها الى فعالية طريقة RPSM لحل هذا النوع من مسائل القيم الابتدائية.

الكلمات المفتاحية: طريقة متسلسلة القوى المتبقية، معادلة كورتيج دي فريس وبرجرز المعدلة الكسرية، معنى كابوتو.

Introduction

In the last decade, the exploration of memory effect, heredity properties and non-local behaviours of mathematical models in various disciplines and fields of science, has gained a tremendous attention among researches. Indeed, the fractional partial differential equations (FPDEs), which is a combination of partial differential equations (PDEs) and fractional calculus, provide a clear description of the interplay between the nonlinearity characteristic and the non-integer order of the derivative involved. Further, wide range of physical phenomena, engineering, finance, and even social sciences with memory are modelled using FPDEs [1,2].
Moreover, numerical and semi-analytical methods were utilized extensively, of late, to construct the approximate solutions of FPDEs, among these methods, the new transform method (ITM) and the residual power series transform method (RPSTM) [3], Laplace residual power series technique [4], the rational non-polynomial splines [5], The Homotopy perturbation transform method (HPTM) and the

Yang transform decomposition method (YTDM) [6], and lastly but not last, the New Local Fractional Mohand–Adomian Decomposition[7]. In study [8], Abu Arqub was the first to establish the RPSM to solve fuzzy differential equations of second order. This method is composed of Taylor's series and the residual error function, afterwards it was developed to a new alogarithm that is effiecintly applied on non-linear higher intial value (IVPs) and boundary value problems (BVPs) to construct power series solutions [9–11].

Many studies have empoyed the RPSM for obtaining the approximate analytical soltions of FPDEs[4,12], El-Ajou et. al. [13] utlized the method to seek the explicit solutions of time-space fractional KdV-burgers equation, on the other hand, the advatage of appling an integral transform with RPSM was estaplished in a recent study by Iqbal et. al. [14] in which they have used it to obtain the semi-analytical solutions to fractional modified KdV equation an coupled Burger's equations along with another well-known Iterative method.

In this research article, we aim to investigate the applicability, and efficiency of the RPSM on constructing the semi-analytical solution of the time-fractional modified Korteweg-de Vries Burger's equation (mKdV-Burger's), in the Caputo' sense, of the form:

$$D_t^{\gamma}\mathcal{Y} + a\mathcal{Y}^2\partial_x\mathcal{Y} + \alpha\,\partial_{xxx}\mathcal{Y} + \beta\partial_{xx}\mathcal{Y} = 0, \qquad x \in R, t > 0, 0 < \gamma \le 1 \tag{1}$$

Subjected to the IC:

$$\mathcal{Y}(x,0) = g_0(x) \tag{2}$$

where $\mathcal{Y}(x,t)$ is the wave function, of time and space, in which $|\mathcal{Y}(x,t)|$ approaches zero, as x and t are both tending to negative infinity .The notations, a, α_1,and β_1 are arbitrary real constants, such that a, α , $\beta \neq 0$, and $\alpha < 0$, to be more specific, a is the coefficient of the nonlinear term, α governs the dispersion, and β is the coefficient of dissipation. Plus, in which $x \in [0,\infty)$ the initial condition function is analytic. The time-fractional mKdV-Burgers is a general frame work for special case by setting $\beta = 0, \alpha = 0, \beta = \alpha = 0, a = \alpha = 0, a = \beta = 0$, and $a = \beta = \alpha = 0$, equation (1) is reduced to the Modified Korteweg-de Vries equation (mKdV), the modified Burgers equation, the nonlinear advection equation, the diffusion equation, the linear dispersive equation, and the advection equation (*respectively*) of time fractional order [15–17].

The current article is outlined in the following way, in the section 1, most influential and fundamental concepts of fractional calculus were provided along with the essential definitions of the method of our consideration. In Section 0, the RPSM was employed on the general form of the time-fractional modified KdV-Burger's equations (1) and (2). A detailed interpretation of the numerical simulation on the effect of the fractional derivative was demonstrated in section 4. Finally, the merits and eligibility of the method are given as a brief conclusion in section 5.

Basic Concepts and Representation

In this section, the essential concepts, definitions, theorems, as well as results used in the study are fully addressed **below** [1,2,9–12].

Definition 1. The Riemann-Liouville (R-L) operator with fractional order $1 > \gamma \ge 0$ of a function g in the space C_μ , $\mu \ge -1$, is defined as:

$$\mathcal{J}^{\gamma} g(t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-\tau)^{\gamma-1} g(\tau)d\tau, \qquad \gamma > 0, t > 0 \tag{3}$$

where $\mathcal{J}^0 g(t) = g(t)$, and $\Gamma(\gamma) = \int_0^\infty \tau^{\gamma-1}\, e^{-\tau} d\tau$.

Definition 2. The fractional derivative of order γ in the Caputo sense, of a function g, is

$$D^{\gamma}\, g(t) = \begin{cases} \mathcal{J}^{n-\gamma} g^{(n)}(t) = \dfrac{1}{\Gamma(\gamma)} \displaystyle\int_0^t (t-\tau)^{(n-\gamma-1)} g^{(n)}(t) d\tau, & t,\tau > 0, n-1 < \gamma \le n \\ \dfrac{d^n g(t)}{dt^n}, & \gamma = n \end{cases} \tag{4}$$

We provide, below, some useful properties on the R-L operator and the Caputo derivative:

1. $\mathcal{J}^{\gamma} C = \frac{c}{\Gamma(\gamma+1)}\, t^{\gamma}$
2. $\mathcal{J}^{\gamma} J^{\beta} g(t) = \mathcal{J}^{\gamma+\beta} g(t) = \mathcal{J}^{\beta}\mathcal{J}^{\gamma} g(t)$
3. $\mathcal{J}^{\alpha} t^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} t^{\alpha+\gamma}$
4. If C a constant $D^{\gamma} C = 0$, .
5. $D^{\gamma} t^m = 0, \quad m < \gamma$
6. $D^{\gamma} t^m = \frac{\Gamma(m+1)}{\Gamma(m+1-\gamma)} t^{m-\gamma} \qquad, m \ge \gamma$.

Definition 3. The Caputo time-fractional operator of order γ of $\mathcal{Y}(x,t)$ is given by,

$$D_t^{\gamma}\mathcal{Y}(x,t) = \partial_t^{\gamma}\mathcal{Y}(x,t) = \frac{1}{\Gamma(m-\gamma)} \int_0^t (t-\nu)^{m-\gamma-1} \frac{\partial^m \mathcal{Y}(x,\nu)}{\partial t^m} d\nu, \quad m-1 < \gamma < m \tag{5}$$

and that $D_t^m \mathcal{Y}(x,t) = \frac{\partial^m \mathcal{Y}(x,t)}{\partial t^m}$, $\quad m \in \mathbb{N}$.

Definition 4. A fractional power series expansion (FPS) at $t = t_0$ is given by,

$$g_0(x) + g_1(x)(t-t_0)^\gamma + g_2(x)(t-t_0)^{2\gamma} + \cdots = \sum_{m=0}^{\infty} g_m(x)(t-t_0)^{m\gamma}$$

$$,0 \le m-1 < \alpha \le m \;\; t \ge t_0, \tag{6}$$

Where $g_m(x)(m = 0,1,2,\dots)$ is the coefficients of the series.

Definition 5. (See [1,12]). The expansion of $\mathcal{Y}(x,t)$ at t_0 of an FPS is given by:

$$\mathcal{Y}(x,t) = \sum_{m=0}^{\infty} g_m(x)(t-t_0)^{m\gamma} = \sum_{m=0}^{\infty} D_t^{m\gamma}\mathcal{Y}(x,t_0)/\Gamma(m\gamma+1)\,(t-t_0)^{m\gamma},$$

$$0 \le m-1 < \gamma \le x \in I, t_0 \le t < t_0 + R \tag{7}$$

where $D_t^{m\gamma}\mathcal{Y}(x,t)$ for , $m = 0,1,2,\dots$. is continuous on the rectangular $I \times (t_0, t_0+R)$, and R is the radius of convergence. The formula in (7) is the Generalized Taylor Series. Note that if $\gamma = 1$,the formula in equation (7) gives the classical Taylor series.

RPSM for the Modified KdV-Burger's Equation

In this section, we are applying the RPSM to the time-fractional modified KdV-Burger's equations (1) and (2). According to El-Ajou *et. al.*[12], and by the use of equation (7), the solution of (1) can be expressed in a series form as:

$$\mathcal{Y}(x,t) = \sum_{m=0}^{\infty} g_m(x)\frac{t^{m\gamma}}{\Gamma(1+m\gamma)} \tag{8}$$

And approximate solution of the k[th] order of equation (8), is:

$$\mathcal{Y}_k(x,t) = \sum_{m=0}^{k} g_m(x)\,\frac{t^{m\gamma}}{\Gamma(1+m\gamma)}, \tag{9}$$

since $\mathcal{Y}(x,t)$ holds the IC equation (2) $\mathcal{Y}(x,0) = g_0(x) = g(x)$, then equation (9) could be rewritten as:

$$\mathcal{Y}_k(x,t) = g(x) + \sum_{m=1}^{k} g_m(x)\,\frac{t^{m\gamma}}{\Gamma(1+m\gamma)}, \tag{10}$$

Where $\quad 0 < \gamma \le 1, \quad x \in I, t \ge 0\,, k = 1,2,3,\dots\,, \;\; (m = 0,1,2,\dots,k)$.

To compute the coefficients $g_m(x)$, the residual function for the KdV-Burger's equations (1) is defined by:

$$\mathcal{R}es(x,t) = \partial_t^\gamma \mathcal{Y}(x,t) + a\mathcal{Y}(x,t)^2\partial_x\mathcal{Y}(x,t) + \alpha\partial_{x,x,x}\mathcal{Y}(x,t) + \beta\partial_{x,x}\mathcal{Y}(x,t) \tag{11}$$

And the k[th] residual function for $k = 1,2,\dots$ is:

$$\mathcal{R}es_k(x,t) = \partial_t^\gamma \mathcal{Y}_k(x,t) + a\mathcal{Y}_k(x,t)^2\partial_x\mathcal{Y}_k(x,t) + \alpha\partial_{x,x,x}\mathcal{Y}_k(x,t) + \beta\partial_{x,x}\mathcal{Y}_k(x,t)\,, \tag{12}$$

In which $\mathcal{R}es(x,t) = 0$ and that, $\mathcal{R}es_k(x,t)$ approaches $\mathcal{R}es(x,t)$ as k tends to infinity, $\forall\, x \in I, t > 0$. Eventually, $D_t^{n\gamma}\mathcal{R}es(x,t) = 0$, $\;D_t^{n\gamma}\mathcal{R}es(x,0) = D_t^{n\gamma}\mathcal{R}es_k(x,0) = 0\;,(n = 0,1,\dots,k)$ (for further detailed explanation see [12,18]).

when $k = 1$ equations (10), and (12) become:

$$\mathcal{Y}_1(x,t) = g(x) + \sum_{m=1}^{1} g_m(x)\,\frac{t^{m\gamma}}{\Gamma(1+m\gamma)} = g(x) + g_1(x)\frac{t^\gamma}{\Gamma(1+\gamma)} \tag{13}$$

And,

$$\mathcal{R}es_1(x,t) = \partial_t^\gamma \mathcal{Y}_1 + a{\mathcal{Y}_1}^2\partial_x\mathcal{Y}_1 + \alpha\partial_{x,x,x}\mathcal{Y}_1 + \beta\partial_{x,x}\mathcal{Y}_1 \tag{12}$$

Substituting equation (13) into $\mathcal{R}es_1(x,0) = 0$, and solving for the unknown $g_1(x)$, to get:

$$g_1(x) = -ag(x)^2 g(x) - \alpha_1 g'''(x) - \beta_1 g''(x) \tag{15}$$

When $k = 2$, we get:

$$\mathcal{Y}_2(x,t) = g(x) + \sum_{m=1}^{2} g_m(x)\,\frac{t^{m\gamma}}{\Gamma(1+m\gamma)} = g(x) + g_1(x)\frac{t^\gamma}{\Gamma(1+\gamma)} + g_2(x)\frac{t^{2\gamma}}{\Gamma(1+2\gamma)} \tag{16}$$

And,

$$\mathcal{R}es_2(x,t) = \partial_t^\gamma \mathcal{Y}_2 + a{\mathcal{Y}_2}^2\partial_x\mathcal{Y}_2 + \alpha\partial_{x,x,x}\mathcal{Y}_2 + \beta\,\partial_{x,x}\mathcal{Y}_2 \tag{17}$$

Substituting equation (16) into $D_t^\gamma\mathcal{R}es_2(x,0) = 0$, we obtain:

$$g_2(x) = -ag(x)g_1(x)g'(x) - a\,g(x)^2 g_1'(x) - \alpha g_1'''(x) - \beta g_1''(x) \tag{18}$$

When $k = 3$, to yield:

$$\mathcal{Y}_3(x,t) = g(x) + \sum_{m=1}^{3} g_m(x)\,\frac{t^{m\gamma}}{\Gamma(1+m\gamma)}$$

$$= g(x) + g_1(x)\frac{t^{\gamma}}{\Gamma(1+\gamma)} + g_2(x)\frac{t^{2\gamma}}{\Gamma(1+2\gamma)} + g_3(x)\frac{t^{3\gamma}}{\Gamma(1+3\gamma)} \tag{19}$$

And the residual function is

$$\mathcal{R}es_3(x,t) = \partial_t^{\gamma}\mathcal{Y}_3 + a\mathcal{Y}_3{}^2\partial_x\mathcal{Y}_3 + \alpha\partial_{x,x,x}\mathcal{Y}_3 + \beta\,\partial_{x,x}\mathcal{Y}_3 \tag{20}$$

By the use of $D_t^{2\gamma}\mathcal{R}es_3 = 0$, the coefficient $g_3(x)$ is given by:

$$g_3(x) = -2ag(x)g_2(x)g'(x) - ag_1(x)\frac{\Gamma(1+2\gamma)}{\Gamma(1+\gamma)^2}\left(g_1(x)g'(x) + 2g(x)g_1'(x)\right)$$

$$-ag^2(x)g_2'(x) - (\beta+\alpha)g_2''(x) \tag{21}$$

And so forth.

Results and discussion

In this section, the 3[nd] term approximated solution of the mKdV-Burger's equation (19) is taken into consideration to investigate the memory effect of the fractional order γ on the wave solution. The parameters used for the numerical simulation are $a = 1,\ \alpha = -0.25, \beta = 1.5$, with the initial condition $,\mathcal{Y}(x,0) = \sqrt{1.5}\,(1+\text{Tanh}(x))$ and the analytical solution for $\gamma = 1$ is $\mathcal{Y}(x,t) = \sqrt{1.5}\,(1+\text{Tanh}(x-4t))$ [15,16].

In Figure 1, and Figure 2 the comparison of analytical solution $\mathcal{Y}(x,t)$ when ($\gamma = 1$) with the first three approximations $\mathcal{Y}_1, \mathcal{Y}_2$ and $\mathcal{Y}_3$ was illustrated in 3D and 2D plots.

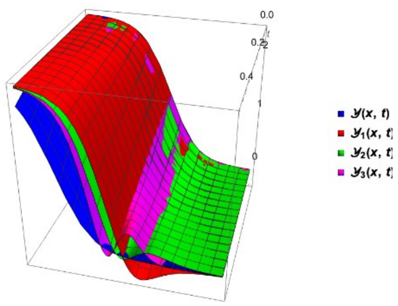

Figure 1 The 3D surface comparison of the analytical solution $\mathcal{Y}(x,t)$ and the approximate solutions $\mathcal{Y}_k(x,t)(k = 1,2,3)$ of the time-fractional modified KdV-Burger's equation of order $\gamma = 1$, for $x \in [-3,3], and\ t \le 0.5$

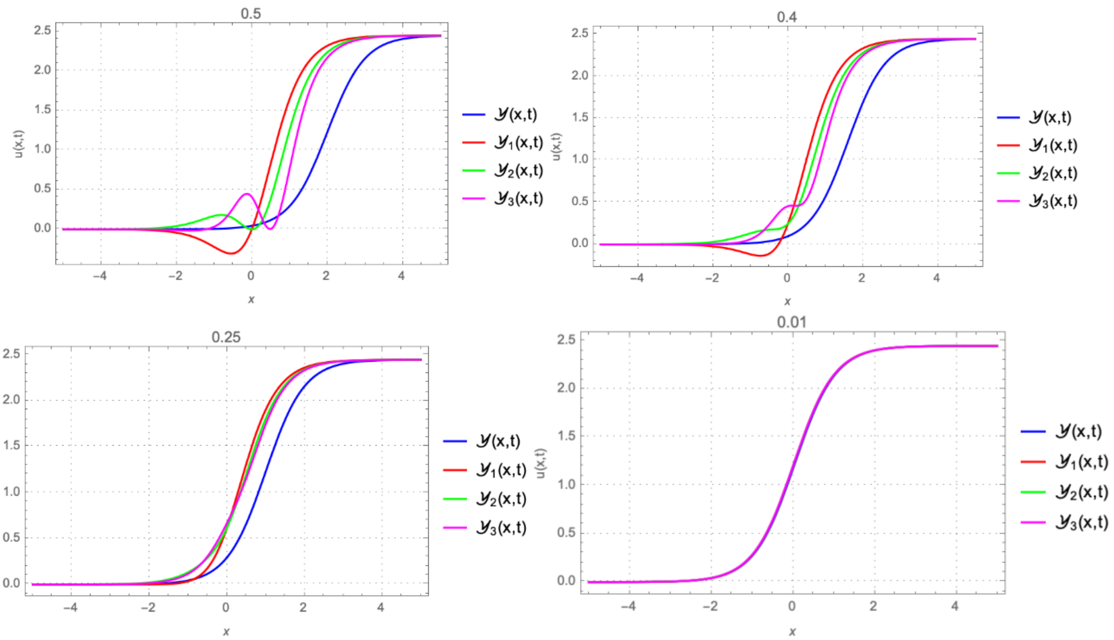

Figure 2 The 2D curves comparison of the analytical solution $\mathcal{Y}(x,t)$ and the approximate solutions $\mathcal{Y}_k(x,t)(k = 1,2,3)$ of the time-fractional modified KdV-Burgers equation of order $\gamma = 1$, for $x \in [-5,5]$, and $t = 0.5, 0.4, 0.25, 0.01$. (*respectively*).

By observation, those graphs demonstrate the convergent of the approximate solutions to the exact solution as the time decrease, in which they meet perfectly as $t \leq 0.01$. In order to further examine the effect of the fractional order γ, Figure 3 exhibits the behavior of the exact solution $u(x,t)$ and the approximate solution $u_3(x,t)$ at time value 0.25, in which the approximate solution in the third subgraph of Figure 2 has its verge on both converging and diverging to the exact one.

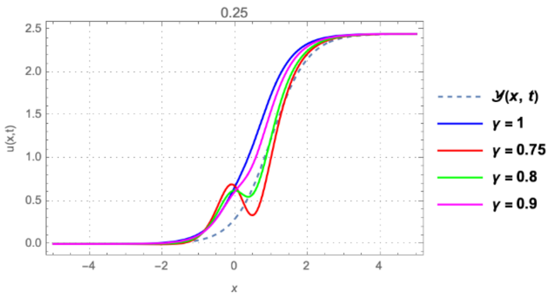

Figure 3 The 2D curve comparison of the analytical solution $\mathcal{Y}(x,t)$ and the approximate solution $\mathcal{Y}_3(x,t)$ when $\gamma = 1$. Along with the effect of the fractional order $\gamma = 0.75, 0.8, 0.9$ on $\mathcal{Y}_3(x,t)$.

It is clearly that in Figure 3 the approximate solution meets the analytical solution at the end of x domain, but shows some wiggles as the space values approaches zero. Also, the time fractional order has the impact on converging of the approximate solution as γ tends to the integer order.

Further, for the purpose of a detailed examination of the RPSM's efficiency, Figure 4 provides the 3D graphical display of the absolute error, observing its enlargement around the grid point $x = 0$ as the time tends to larger values. The 2D graph emphasizes this observation and proves it.

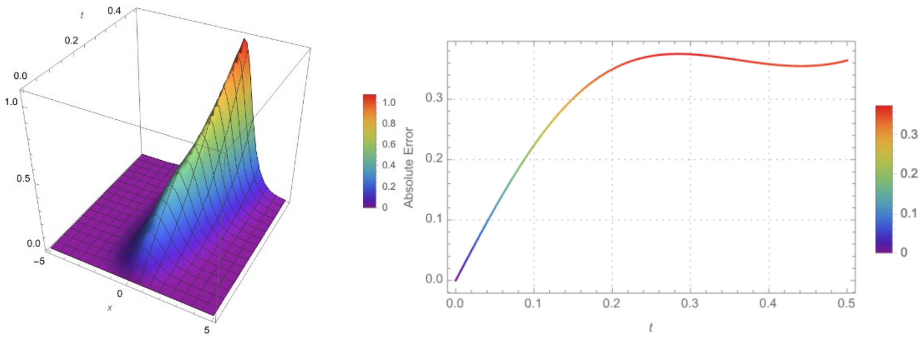

Figure 4 The 3D surface absolute error surface for $\gamma = 1$, and $x \in [-5,5], t \leq 0.5$, and the 2D absolute error for $x = 0$, $t \in [0,0.5]$.

Table 1 summarizes a numerical comparison for time $t \leq 0.2$ and $x = 5$ where the absolute error at its minimum as shown the in the left subgraph in Figure 4. The absolute error values affirm that as time increases the error increase and the series solution diverges.

Table 1 The comparison of the analytical solution $\mathcal{Y}(x,t)$ and $\mathcal{Y}_3(x,t)$ for $\gamma = 1$, $t \leq 0.2$, and $x = 5$.

t	$u(x,t)$	$u_3(x,t)$	Absolute Error
0	2.44938	2.44938	0.
0.05	2.44932	2.44935	3.01× 10-5
0.1	2.44924	2.44932	8.17× 10-5
0.15	2.44912	2.44929	1.67× 10-4
0.2	2.44894	2.44924	3.05× 10-4

Conclusion

In this study, the method of RPS was successfully employed to obtain the approximate solution of the time-fractional modified KdV-Burger's equation. The approximate solution was presented in a form of a power series method that converges rapidly despite its radius of convergence and the complexity of the symbolic computations. The numerical investigation emphasizes the efficiency, applicability, and generality of applying the RPSM for such IVPs.

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