Afro-Asian Journal of Scientific Research (AAJSR)
المجلة الأفروآسيوية للبحث العلمي
E-ISSN: 2959-6505
Volume 3, Issue 4, 2025
Page No: 253-258
Website: https://aajsr.com/index.php/aajsr/index
SJIFactor 2024: 5.028 ISI 2025: 0.915 معامل التأثير العربي (AIF) 2025: 0.76

A New Approach for Solving Linear Programming Problems with Uncertainty Coefficients

Abdussalam Mohamed Khalifa[1*], F. M. Hejaj[2]
[1,2]Mathematics Department, Faculty of Education, Azzaytuna University, Tarhuna, Libya

طريقة جديدة لحل مشكلة البرمجة الخطية ذات المعاملات غير يقينية

عبد السلام محمد خليفة[1*]، فادية مفتاح حجاج[2]
[1،2]قسم الرياضيات، كلية التربية، جامعة الزيتونة، ترهونة، ليبيا

*Corresponding author: abmaabkhalifa@gmail.com

Received: October 11, 2025 | Accepted: December 22, 2025 | Published: December 30, 2025

Abstract:
In this paper, this algorithmic solution based on rough interval coefficients for uncertainty parameters is developed using the lower and upper interval (LILP) and (UILP) approaches. We proposed method addresses the uncertainty (rough interval) Programming (RIP) problem, in which the coefficients of the objective function and constraints are characterized as uncertainty intervals. An algorithm gives an efficient decision support tool structure for solving linear programming problems characterized by uncertain data. It is shown that the solutions of the related crisp problems, namely the Upper Crisp Interval linear programming Problem (UIP(λ)) and the Lower Crisp Interval linear programming Problem (LIP(λ)), The solution of the (UIP(λ)) and (LIP(λ)) are depends on the value of form limits of the interval solution of the RIP problem and its optimal solution, the solution of the (UIP(λ)) and (LIP(λ)) are depends on the value (λ) which decision maker. The applicability of the proposed approach is verified through a numerical example.

Keywords: Linear Programming Problem, Rough Interval, Optimal Solution.

الملخص:
في هذه البحث، تم تطوير نهج جديد لحل مشكلة البرمجة الخطية بمعاملات غير يقينية، وذلك باستخدام تجزئة الفترة الغامضة إلى فترتين أحدهم للفترة العليا والأخرى للفترة الدنيا للعدد الغامض، ثم بعد ذلك يتم إنشاء مشكلتين وهما مشكلة البرمجة الخطية ذات المعاملات العليا للعدد الغامض والأخرى مشكلة البرمجة الخطية ذات المعاملات الدنيا للعدد الغامض لمشكلة البرمجة الخطية بمعاملات غير يقينية. وهذا النهج المقترح يوفر قيمة يقينية يحددها متخذ القرار وفقًا لتفصيلاته، لكل من مشكلة البرمجة الخطية ذات الفترة العليا ومشكلة البرمجة الخطية ذات الفترة الدنيا. وبعد حل تلك المشكلتين باستخدام طريقة السمبليكس، ونشكل الحل الأمثل لمشكلة البرمجة الخطية ذات المعاملات الغامضة من حل تلك المشكلتين العليا والدنيا ويكون في صورة فترة، حيث القيمة العليا للفترة هي الحل الأمثل لمشكلة البرمجة الخطية ذات المعاملات العليا للعد الغامض والقيمة الدنيا هي الحل الأمثل لمشكلة البرمجة الخطية ذات المعاملات الدنيا للعدد الغامض، وقد تم التحقق من كفاءة وقابلية تطبيق هذا المقترح من خلال مثال عددي توضيحي.

الكلمات المفتاحية: مسألة البرمجة الخطية، فترة تقريبية، الحل الأمثل.

Introduction:

The theory of uncertainty parameters, originally introduced by (Pawlak, 1982) provides a mathematical framework for handling indeterminacy, incompleteness and uncertainty in data . Rough interval theory has since been known as an efficient tool for modeling imprecise information without requiring preliminary and additional information about data .

To further develop the capability of indeterminacy modeling, (Dubois & Prade, 1990) combined the definitions of rough interval and fuzzy sets, leading to the development of fuzzy rough interval. These mixture models have proven efficient in instead of imprecision inherent in real data. therefore, many extensions and generalizations of rough interval and fuzzy sets, have been proposed based on logical different operators and approximation mechanisms in the direction of improve their computational power (Klir & Yuan, 2008).

In parallel, the indeterminacy in optimization problems it gets a lot of attention, particularly of linear programming problems models with vague parameters. More than on study have addressed optimization problems with rough interval and fuzzy coefficients as a means of capturing indeterminacy of objective functions and constraints (Chinneck &Ramadan, 2000: Moore,1979). However, when the indeterminacy is represented by rough intervals or fuzzy classical solution approaches become insufficient.

In this paper, we propose a novel algorithmic approach to decide the optimal solution of uncertainty linear programming problems in which the coefficients of the problem are expressed as rough intervals in objective function and constraints. The planned method depends on the construction of lower interval model and upper interval model that capture the uncertainty in the problem parameters. This approach can give out as a practical and reliable decision support tool for addressing a wide range of optimization problems linking uncertainty data. Also, it is demonstrated that the optimal solutions obtained from the LILPP and UILPP bound the feasible solution set and the optimal objective value of the RILPP.

Problem formulation:

Consider the rough interval problem (RIP):

$$\min f^R = \sum_{j=1}^{n} \left[[c_j{}^{LU}, c_j^{UU}] : [c_j{}^{LL}, c_j^{UL}] \right] x_j^R$$

such that

$$\sum_{j=1}^{n} \left[a_{ij}{}^{LU}, a_{ij}^{UU} \right] : \left[a_{ij}{}^{LL}, a_{ij}^{UL} \right] x_j^R \leq \left[b^{LU}, b_{ij}^{UU} \right] : \left[b_{ij}{}^{LL}, b_{ij}^{UL} \right] \quad (1)$$

$$x_j^R \geq 0 \quad (2)$$

$i = 1,2, \dots m$ and $j = 1,2, \dots n$

Definition 1: (Feasible solution)

An element x satisfying all the constraints (1) and (2) arecalled a feasible solution (Pandian & Natarajan, 2010a).

Definition 2:(optimal Soltion)

Let $f(x)$ denote the objective functon. A feasible solution x^* is called an optimal solution if it gives the minimum value of $f(x)$ in a minimization problem or the maximum value of $f(x)$) in a maximization problem, in all feasible solutions (Moore, 1979)

Theorem 1: *The optimal solution of the RIP, we get from the solutions of the two* problems:

(1)

$$UIP \quad : \max f^{IU} = \sum_{j=1}^{n} c_j^{IU} \, x_j^{IU}$$

Subject:

$$\sum_{i=1}^{n} a_{ij}^{IU} x_I^{IU} \leq b_{ij}^{IU},$$
$$x_i^{IU} \geq 0$$

We get the optimal soltion x_i^{*IU} for the UIP.

(2)

$$LIP : \max f^{IL} = \sum_{I=1}^{n} c_i^{IL} \, x_i^{IL}$$

Subject:

$$\sum_{i=1}^{n} a_{ij}^{IL} x_I^{IL} \leq b_{ij}^{IL}, \text{ j=1,2,....,m}$$
$$x_i^{IL} \geq 0$$

We obtain the optimal solution x_i^{*IL} for the UIP.

proof:

Let for $s = \{[x_j^{LU}, x_j^{UU}] : [x_j{}^{LL}, x_j^{UL}]\}$, for all j } be a rough interval feasible solution RIP

Therefore, { $[x_j^{LU}, x_j^{UU}]$, for all j }, $\{[x_j{}^{LL}, x_j^{UL}]\}$, for all j are feasible solutions of the problems $UIP\ and\ LIP$.

Now, since x_λ^{*U}, for all $\lambda \in [c_j{}^{LU}, c_j{}^{UU}]$ and x_λ^{*L}, for all $\lambda \in [c_j{}^{LL}, c_j{}^{UL}]$}, are optimal solutions of $UIP\ and\ LIP$,we have

$$\sum_{j=1}^{n} c_j^{IU}(\lambda)\, x_j^{*IU} \le \sum_{j=1}^{n} c(\lambda)_j^{IU}\, x_j^{IU}$$

$$\sum_{i=1}^{n} a_{ij}^{IU}\,(\lambda) x *_I^{IU} \le b_{ij}^{IU}(\lambda),$$

And

$$\sum_{j=1}^{n} c_j^{IL}(\lambda)\, x_j^{*IL} \le \sum_{j=1}^{n} c(\lambda)_j^{IL}\, x_j^{IL} \text{ and}$$

$$\sum_{i=1}^{n} a_{ij}^{IL} \le b_{ij}^{IL}(\lambda)$$

With the condition

$$x_{ij}^{*IL}(\lambda) \le x_{ij}^{*IU}(\lambda)$$

This implies that to

$$[\sum_{j=1}^{n} c_j^{IL}(\lambda) x_j^{*IL} : \sum_{j=1}^{n} c_j^{IU}(\lambda)\, x_j^{*IU}] \le [\sum_{j=1}^{n} c(\lambda)_j^{IL}\, x_j^{IU} : \sum_{j=1}^{n} c(\lambda)_j^{IU}\, x_j^{IU}]$$

we can conclude that the set $[x_{j(\lambda),}^{*L} x_{j(\lambda),}^{*U}]$ for all λ is a feasible solution of $RIP(\lambda)$.

Thus, the set of intervals $[x_{j(\lambda),}^{*L} x_{j(\lambda),}^{*U}]$ is an interval optimal solution of the proble $RIP(\lambda)$.

Algorithm Solution:

This algorithm for finding an optimal solution for RIP. The algorithm for the procedure is as follows.

1. Consider the rough interval linear programming problem $RILP$.
2. Formulate the UILP corresponding to the given RILP.
3. convert the $UILP$ to the crisp upper linear programming problem $CULP(\lambda)$.
4. Finding the optimal solution inside a point in interval of coefficients of **.**
5. We choose a parameter value $\lambda = \lambda_0 \in [0,1]$, Then each interval coefficient of CULP is transformed into a crisp value as follows :

$$a_{ij(\lambda)} = a_{ij}^L + \lambda(a_{ij}^U - a_{ij}^L),\ b_{ij(\lambda)} = b_{ij}^L + \lambda(b_{ij}^U - b_{ij}^L) \text{ and } c_{ij(\lambda)} = c_{ij}^L + \lambda(c_{ij}^U - c_{ij}^L)$$

6. we get the Crisp Linear Programming Model at an Interior Point as formulas:

$$CUP(\lambda) \qquad maximize\ f(\lambda_0) = \sum_{j=1}^{n} c_j x_j$$

Such that:

$$\sum_{j=1}^{n} a_{ij(\lambda_0)}\, x_j \le b_i(\lambda_0)$$

$$x_j \ \ge\ 0, i = 1,2, \dots, m$$

7. Solve the $CUP(\lambda_0)$ using the Simplex method . Let $x_j^{*U}(\lambda_0)$ denote an optimal solution of fo $CUP(\lambda_0)$.
8. Formulate the LIP corresponding to the given RILP.
9. convert the $LILP$ to the crisp upper linear programming problem $CLLP(\lambda)$.
10. Finding the optimal solution inside a point in interval of coefficients of $CLLP(\lambda_0)$.
11. We choose a parameter value $\lambda = \lambda_0 \in [0,1]$, Then each interval coefficient of CLLP is transformed into a crisp value as follows:

$$a_{ij(\lambda)} = a_{ij}^L + \lambda(a_{ij}^U - a_{ij}^L),\ b_{ij(\lambda)} = b_{ij}^L + \lambda(b_{ij}^U - b_{ij}^L) \text{ and } c_{ij(\lambda)} = c_{ij}^L + \lambda(c_{ij}^U - c_{ij}^L$$

12. we get the Crisp Linear Programming Model at an Interior Point as formulas:

$$CLP(\lambda)\ \ maximize\ f(\lambda_0) = \sum_{j=1}^{n} c_j x_j$$

Such that:

$$\sum_{j=1}^{n} a_{ij(\lambda_0)}\, x_j \le b_i(\lambda_0)$$

$$x_j \ \ge\ 0, i = 1,2, \dots, m$$

13. Solve the $CLP(\lambda_0)$ using the Simplex method . Let $x_j^{*L}(\lambda_0)$ denote an optimal solution of fo $CLP(\lambda_0)$.
14. The optimal solution of the given RIP is:

$$x_j{}^*(\lambda_0) = (x_j^{*L}(\lambda_0), x_j^U(\lambda_0)), \qquad \lambda_0 \in [0,1]$$

$$\text{And } f_j{}^*(\lambda_0) = [f_j^{*L}(\lambda_0), f_j^U(\lambda_0)], \qquad \lambda_0 \in [0,1]$$

This example explains how the above algorithm systematically transforms the input into the final output optimal solution.

Example (1):

$min\ \ f\ = \sum_{j=1}^{3} c_j^r x_j$

such that:

$$\sum_{j=1}^{3} a_{ij}x_j \le b_i \qquad i = 1,2,3,$$

$$x_j^R \ge 0\,, \qquad j = 1,2,3$$

assume that the rough interval characterized as:

c_j^r =(]59.5,60.5]:[59,61] [49.5,50.50.5]:[49,51] [29.5,30.5]:[29,31])

$$a_{ij}^r = \begin{pmatrix} [2.5,3.5]:[2,4] & [1.5,2.5]:[1,3] & [0.5,1.5]:[0,2] \\ [1.5,2.5]:[1,3] & [2.5,3.5]:[2,4] & [1.5,2.5]:[1,3] \\ [0.5,1.5]:[0,2] & [0.5,1.5]:[0,2] & [3.5,4.5]:[3,5] \end{pmatrix}$$

$$b_i^r = \begin{pmatrix} [239.5,240.5]:[239,241] \\ [199.5,200.5]:[199,201] \\ [179.5,180.5]:[179,181] \end{pmatrix}$$

Hence, the above RIP problem can be formulated as follows:

$UILP \qquad \min f^{UI} = \sum_{j=1}^{3} c_j^{UI} x_j^{UI}$

Such that :

$$\sum_{j=1}^{3} a_{ij}^{UI} x_j^{UI} \le b_J^{UI},\ \ i = 1,2,3$$

$$x_j^{UI} \ge 0 \quad i = 1,2,\ldots,n$$

where

$$c_j^{UI} = ([59,61] \quad [49,51] \quad [29,31])$$

$$a_{ij}^{UI} = \begin{pmatrix} [2,4] & [1,3] & [0,2] \\ [1,3] & [2,4] & [1,3] \\ [0,2] & [0,2] & [3,5] \end{pmatrix}$$

$$b_{ij}^{UI} = \begin{pmatrix} [239,241] \\ [199,201] \\ [179,181] \end{pmatrix}$$

Also, the RIP problem can be expressed in LI form as follows

$LIP \qquad \min f^{LI} = \sum_{j=1}^{3} c_j^{LI} x_j^{LI}$

such that :

$$\sum_{j=1}^{3} a_{ij}^{LI} x_J^{LI} \le b_J^{LI},\ \ i = 1,2,3$$

$$x_j^{LI} \ge 0 \quad i = 1,2,\ldots,n$$

Where

$$c_j^{LI} = ([59.5,60.5] \quad [49.5,50.5] \quad [29.5,30.5])$$

$$a_{ij}^{LI} = \begin{pmatrix} [2.5,3.5] & [1.5,2.5] & [0.5,1.5] \\ [1.5,2.5] & [2.5,3.5] & [1.5,2.5] \\ [0.5,1.5] & [0.5,1.5] & [3.5,4.5] \end{pmatrix}$$

$$b_i^{LI} = \begin{pmatrix} [239.5,240.5] \\ [199.5,200.5] \\ [179.5,180.5] \end{pmatrix}$$

Now, in the crisp formulation of the two problems UIP and LIP, and assuming that the paremeter takes the value $\lambda_0 = 0.3$. the variables λ_0 is calculated as follows:

We find the solution to the problem based on the parameter λ_0. defined by the decision-maker $\lambda_0 = 0.3$.

$a_{ij(\lambda)} = a_{ij}^L + 0.3(a_{ij}^U - a_{ij}^L)$, $b_{ij(\lambda)} = b_{ij}^L + 0.3(b_{ij}^U - b_{ij}^L)$ and $c_{ij(\lambda)} = c_{ij}^L + 0.3(c_{ij}^U - c_{ij}^L)$

The $UIP(\lambda)$ as :

$UIP(\lambda_0)$ $\min f^{UI} = \sum_{j=1}^{3} c_j^{UI}(\lambda_0)x_j^{UI}$

Such that:

$$\sum_{j=1}^{3} a_{ij}^{UI}(\lambda_0)x_J^{UI} \le b_J^{LI}(\lambda_0), \quad i = 1,2,3$$

$$x_j^{UI} \ge 0 \quad i = 1,2,\dots,n$$

Then, we get

$$c_i^U(\lambda_0) = (59.6 \quad 49.6 \quad 29.6)$$

$$\mathrm{a}_{\mathrm{ij}}^{\mathrm{U}}(\lambda_0) = \begin{pmatrix} 2.6 & 1.6 & 0.6 \\ 1.6 & 2.6 & 1.6 \\ 0.6 & 0.6 & 3.6 \end{pmatrix}$$

$$b_i^U = \begin{pmatrix} 239.6 \\ 199.6 \\ 179.6 \end{pmatrix}$$

By solving the $UIP(\lambda_0)$ problem, we obtain the following solution.

$x_1^{*U}(\lambda_0) = 80.75$, $x_2^{*U}(\lambda_0) = 5.188$ and $x_3^{*U}(\lambda_0) = 35.565$,

$$f_{max}^{*U}(\lambda_0) = 6122$$

For the lower problem

$LIP(\lambda_0)$ $\min f^{LI} = \sum_{j=1}^{3} c_j^{LI}(\lambda_0)x_j^{LI}$

Such that

$$\sum_{j=1}^{3} a_{ij}^{LI}(\lambda_0)x_J^{LI} \le b_J^{LI}(\lambda_0), \quad i = 1,2,3$$

$$x_j^{LI} \ge 0 \quad i = 1,2,\dots,n$$

Then, we get

$$c_i^{LI}(\lambda_0) = (59.8 \quad 49.8 \quad 29.8)$$

$$\mathrm{a}_{\mathrm{ij}}^{\mathrm{LI}}(\lambda_0) = \begin{pmatrix} 2.8 & 1.8 & 0.8 \\ 1.8 & 2.8 & 1.8 \\ 0.8 & 0.8 & 3.8 \end{pmatrix}$$

$$b_i^{LI} = \begin{pmatrix} 239.8 \\ 199.8 \\ 179.8 \end{pmatrix}$$

By solving the $LIP(\lambda_0)$ problem, we obtain the following solution.

$$x_1^{*LI}(\lambda_0) = 74.493 \quad x_2^{*LI}(\lambda_0) = 3.623, \quad x_3^{*LI}(\lambda_0) = 30.870$$

$f^{*LI}{}_{max(\lambda_0)} = 5555$

We optain the optimal solution

$x_1^{*RIP}(\lambda_0) = [74.493, 80.753]$ $x_2^{*RIP}(\lambda_0) = [3.623, 5.188]$ $x_3^{*RIP}(\lambda_0) = [30.870, 35.565]$,

$f^{*LI}{}_{max(\lambda_0)} = [5555, 6122]$

Conclusion:

In this study, We are working on a solution to the Rough Interval Programming Problem was developed based on problem decomposition and interval analysis. The proposed approach begins via decomposing the rough interval coefficients in objective function and constraints of the problem into two deterministic models sub problems an $\mathrm{UIP}(\lambda)$ and a $\mathrm{LIP}(\lambda)$, Which respectively represent the internal and external periods of uncertain transactions.

The an $\mathrm{UIP}(\lambda)$ and a $\mathrm{LIP}(\lambda)$are solved to obtain their corresponding optimal solutions. These solutions provide estimates of the possible area of the solution and the value of the optimal solution with uncertainty coefficients in problem.

To further capture the of the solution within the rough interval range, an interval sampling using this strategy, for each parameter within the interval of objective function and constraints of problem, intermediate values of interval are generated by the relations of $a\_ij(\lambda) = a_{ij}^{L} + 0.3(a_{ij}^{U} - a_{ij}^{L}), b\_ij(\lambda) = b_{ij}^{L} + 0.3\left(b_{ij}^{U} - b_{ij}^{L}\right)$, and $c_{ij}(\lambda) = c_{ij}^{L} + 0.3(c_{ij}^{L} - c_{ij}^{L})$. When λ is determined by the decision-maker.

Finally, the optimal solutions obtained from the $\mathrm{UIP}(\lambda)$ and $\mathrm{LIP}(\lambda)$ problems, together with the sampled intermediate solutions, are integrated to construct a final optimal solution of RIP that efficiently reflects the full range of rough interval. The proposed methodology provides a reliable and computationally efficient decision-support tool for LPP involving uncertainty parameters.

References:

1. Chinneck W. J. and Ramadan J.W. (2000) . Linear programming with interval coefficients, Journal of the Operational Research Society, 51(2) 209 - 220.
2. Dubois D., Prade H. (1990). Rough fuzzy sets and fuzzy rough sets, International Journal of General Systems 17 (2) 191 - 209.
3. George J.Klir and Bo Yuan. (2008). Fuzzy sets and fuzzy logic: Theory and Applications , Prentice-Hall,.
4. He Q., Wu C.X., Chen D.G., Zhao S.Y. (2011). Fuzzy rough set based attribute reduction for information systems with fuzzy decisions, Knowledge-Based Systems 24 (5) 689 - 696.
5. Moore E. R., Method and applications of interval analysis, SLAM, Philadelphia, PA, 1979. .
6. Pandian P. and Natarajan G. (2010a). A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems, Applied Mathematical Sciences, 4(1), 79 - 90.
7. Pandian P. and Natarajan G. (2010b). A new method for finding an optimal solution of fully interval integer transportation problems, Applied Mathematical Sciences, 4, (75), 1819 - 1830.
8. Pawlak Z. (1982). Rough sets, International Journal of Computer Information Sciences 11, (5), 341 - 356.
9. Sengupta A. and Pal T.K. (2003). Interval-valued transportation problem with multiple penalty factors, VU Journal of Physical Sciences , 9, 71 - 81.
10. Tong S. (1994). Interval number and fuzzy number linear programming, Fuzzy sets and systems, 66(3), 301 - 306.