Afro-Asian Journal of Scientific Research (AAJSR)
المجلة الأفروآسيوية للبحث العلمي
E-ISSN: 2959-6505
Volume 4, Issue 1, 2026
Page No: 89-98
Website: https://aajsr.com/index.php/aajsr/index
SJIFactor 2024: 5.028 ISI 2025: 0.915 معامل التأثير العربي (AIF) 2025: 0.76

Applying Hybrid ARIMA-GARCH Models with Three Error Distributions for Modeling Oil Prices in Libya and Algeria

Rabia Awidan*
Department of Statistics, Faculty of Science, University of Tripoli, Tripoli, Libya

تطبيق نماذج *ARIMA-GARCH* الهجينة بثلاثة توزيعات للأخطاء لنمذجة أسعار النفط في ليبيا والجزائر

ربيعة عويدان *
قسم الإحصاء، كلية العلوم، جامعة طرابلس، طرابلس، ليبيا

*Corresponding author: Rabiaa.awedan@uot.edu.ly

Received: December 09, 2025	Accepted: January 21, 2026	Published: January 29, 2026

Abstract:
This study analyzes the statistical properties of monthly domestic crude oil prices in Libya and Algeria from January 1983 to June 2025, aiming to model and forecast price dynamics. For modeling purposes, hybrid models combining the Autoregressive Integrated Moving Average (ARIMA) model and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model with normal, Student-t, and GED error distributions are employed. Series stationarity is assessed using ADF tests, and model selection uses AIC and SIC criteria, with diagnostic checks for residual including Ljung-Box tests, ARCH effects, and normality assessments. Results indicate that the prices series are non-stationary, while the log-differenced series are stationary. Among candidate ARIMA models, ARIMA(0,1,1) and ARIMA(1,1,1) provide the best fit. Including conditional heteroskedasticity, ARIMA-GARCH(1,0) with Student-t distribution produces stable residuals and significant parameters estimates. Out-of-sample evaluation using RMSE and MAE identifies ARIMA(1,1,1)-GARCH(1,0)-t as the most accurate forecasting model. These findings highlight the importance of heavy-tailed error distributions and volatility clustering in modeling domestic oil price dynamics, and demonstrating that the ARIMA-GARCH models provide reliable short-term forecasting efficiency, and contribute to supporting economic and policymakers in the two countries under study.

Keywords: ARIMA Models, GARCH Models, Time Series Analysis, Crude Oil Prices.

الملخص:
هذه الدراسة تحلل الخصائص الإحصائية لأسعار النفط الخام المحلية الشهرية في ليبيا والجزائر من يناير 1983 إلى يونيو 2025، بهدف النمذجة والتنبؤ بديناميكيات الأسعار. لأغراض النمذجة، تم استخدام نماذج هجينة تجمع بين نموذج الإنحدار الذاتي والمتوسطات المتحركة المتكاملة (ARIMA) ونموذج التباين الشرطي غير المتجانس الذاتي المعمم (GARCH) مع توزيعات الأخطاء، الطبيعي وStudent-t وGED. تم تقييم استقرار السلاسل باستخدام اختبارات ADF، وتم اختيار النموذج باستخدام معايير AICوSIC، مع إجراء فحوصات تشخيصية للبواقي تشمل اختبارات Ljung-Box، وتأثيراتARCH، واختبارات التوزيع الطبيعي. تشير النتائج إلى أن سلاسل الأسعار غير مستقرة، بينما السلاسل بعد الفرق الأول لسلاسل اللوغاريتمات مستقرة. ومن بين نماذج ARIMA المرشحة، يوفر كل من ARIMA (0,1,1) وARIMA (1,1,1) أفضل تطابق. مع تضمين التباين الشرطي غير المتجانس يتيح نموذج ARIMA-GARCH (1,0) مع توزيع Student-t بواقي مستقرة وتقديرات للمعالم ذات دلالة إحصائية. ويشير التقييم خارج العينة باستخدام

جذر متوسط مربع الخطأ (RMSE) ومتوسط الخطأ المطلق (MAE) إلى أن نموذج ARIMA (1,1,1) – GARCH (1,0)-t هو نموذج التنبؤ الأكثر دقة. تبرز هذه النتائج أهمية توزيعات الخطأ ذات الذيول السميكة وتكتل التقلبات في نمذجة ديناميكات أسعار النفط المحلية، وتؤكد أن نماذج ARIMA-GARCH توفر كفاءة موثوقة للتنبؤ قصير الأجل، وتسهم في دعم صانعي السياسات الاقتصادية والمالية في البلدين قيد الدراسة.

الكلمات المفتاحية: نماذج ARIMA، نماذج GARCH، تحليل السلاسل الزمنية، أسعار النفط الخام.

Introduction:

Crude oil is a significant energy resource, plays a pivotal role in shaping economic and political condition at both the regional and international levels, with oil prices serving as a key indicator of economic activity. Since the 1973 oil crisis, energy and oil prices have exhibited higher changes than other commodities [1], affecting investor decisions and risk management practices. Consequently, governments and investors are paying increasing attention to analyzing oil price dynamics due to their direct impact on economic policy formulation and investment decision-making [2-4]. In general, oil prices modeling and forecasting have received increasing global attention, with studies focusing on development techniques and quantitative methods, particularly time series models, to represent price dynamics and volatility [5].

Time series models are classified as linear and nonlinear depending on the nature of the relation between the current and past values of the variable, models considered linear if the relationship is linear and non-linear otherwise [6]. In this context, the ARIMA models proposed by Box and Jenkins [7] are one of the most common linear time series models and have been widely applied due to their ability in modeling the dynamic behavior of conditional mean, especially in the case of non-stationary series after differencing [8]. Although the ARIMA models provide good efficiency in short-term forecasting [9,10], but their ability to handle complex features of financial data (e.g., oil price), such as nonlinearity, heterogeneity of variance, high volatility (volatility clustering), and excessive kurtosis remains limited [11]. Therefore, models that explicitly account for these characteristics provide more accurate representations and better predictive performance [12]. In light of these limitations, the time series analysis literature has proposed different categories of non-linear models, including the Autoregressive Conditional Heteroscedasticity (ARCH) model introduced by Engel [13] and the Generalized ARCH (GARCH) model introduced by Bollerslev [14], which allows for modeling conditional variance, or time-varying volatility by incorporating the effects of past volatility into the equation for calculating current volatility. Moreover, ARCH and GARCH models are affective and widely used in analyzing the conditional variance (volatility) especially in the analysis of financial time series data [15]. Hence, combining ARIMA for modeling the conditional mean with GARCH for modeling the conditional variance provides a comprehensive structure for capturing linear and nonlinear behavior, making it particularly suitable for modeling and forecasting of financial data.

Statement of the Problem:

Despite the significance of domestic oil prices, a literature review reveals a lack of studies modeling these prices in developing oil-producing countries compared to developed countries. This study seeks to address this gap through an updated empirical analysis of domestic oil prices in both Libya and Algeria, taking into account the complex statistical features of these time series, such as heterogeneity of variance, volatility clustering, and excessive kurtosis. To achieve that, the study combines the ARIMA models and GARCH models assuming three error distributions for providing a more accurate modeling and improving the predictive power.

The Study Objectives:

The main objective of this study is to analyze the statistical characteristics of domestic oil prices in Libya and Algeria through the following specific objectives:

1. To model the conditional mean of the time series using appropriate ARIMA models.
2. To model the conditional variance of domestic oil prices using GARCH models with alternative error distributions.
3. To evaluate and compare the out-of-sample forecasting accuracy of ARIMA-GARCH models based on standard error measures.

Limitations:

This study is limited to the analysis and modeling of domestic oil prices in Libya and Algeria, two developing oil-producing countries and important members of the Organization of the Petroleum Exporting Countries (OPEC) [16]. The focus on these two countries reflects their comparable economic structures and heavy dependence on oil revenues. Moreover, the analysis is conducted using monthly oil prices data, consistent with previous empirical studies (e.g., [11,17]).

Literature Review:

This section reviews the most major methods used in the literature on oil price analysis. According to Frey et al. [5], these methods can be categorized into three main approaches: time series analysis, economic models, and artificial intelligence techniques. However, this review focuses on time series analysis, paying particular attention to statistical models used to describe and forecast these series. Several studies have investigated the modeling and forecasting of oil prices using time series models, with significant variations in the nature of the data used, its time range, and the statistical models employed. For instance:

Xie et al. [11] explored the forecasting of the monthly price of West Texas Intermediate (WTI) crude oil during 1970-2003 using the ARIMA (1,1,0) model. The results showed that while the linear model is capable of describing the linear structure of the series, it lacks the flexibility to capture nonlinear behavior and complex price dynamics. Moreover, this finding highlights the limitation of ARIMA models when dealing with financial data characterized by high volatility and instability in variance. In this context, subsequent studies have focused on integrating conditional mean models with conditional variance models. Mohammadi and Su [18], evaluated the effectiveness of the ARIMA-GARCH hybrid model, including GARCH, APARCH, EGARCH, and FIGARCH models, for modeling weekly prices in 11 international markets during January 1997 until October 2009, assuming a skewed Student-*t* distribution of errors. The results showed that the APARCH model outperformed the GARCH model in most cases, highlighting the importance of modeling variance asymmetry and high kurtosis in financial data. These results confirm that the characterization of conditional variance leads to better predictive performance compared to linear models. In the study by Yazizet et al. [10], the ARIMA(1,2,1) and GARCH(1,1) models were evaluated for forecasting the daily price of WTI over the period from 1986 to 2009 using out-of-sample. The results showed that the ARIMA model outperformed historical-prices-based forecasts, while the GARCH model excelled in capturing conditional variance and characterizing volatility in the data. In another comparative study, Ahmed and Shabri [19] applied three techniques to forecast WTI crude oil prices during the period 1986-2006, including ARIMA and GARCH models and a Supporting Vector Machine (SVM). The results based on the MAE and RMSE criteria, indicated that the ARIMA model outperformed the GARCH model in terms of forecasting accuracy.

Within the framework of univariate models, Tularam and Saeed [20] compared ARIMA, Holt-Winters and exponential smoothing models to predict monthly oil price between October 2015 and March 2016. The results concluded that the ARIMA (2,1,2) model provided the best predictive performance compared with the other models. On the other hand, Herrera et al. [21] focused on comparing a set of GARCH models with different error distributions to forecast daily oil prices of WTI. The results confirmed that the Student's t distribution was the most appropriate due to the high kurtosis of the data. The EGARCH (1,1) model also showed better performance in medium-term forecasts, while the GARCH (1,1) and RiskMetrics models performed well in short-term forecasting. In an application to local prices, Shah and Thaker [22] examined the forecasting of monthly oil prices in India from April 2000 to October 2023 using the ARIMA-GARCH hybrid mode. The findings showed that the ARIMA (4,2,0)-GARCH (0,3) model was the most suitable compared to other models.

Previous studies indicate the limitation of linear models with constant variance in representing the statistical characteristics of oil prices, especially in high of volatility and nonlinear behavior. Despite the prevalence of GARCH models and their derivatives, the selection of the optimal model remains depends on data features, time frequency, error distributions, and forecast horizon, without a clear consensus in the literature [12,23]. Therefore, the need for comparative applied studies on local data is evident which is what the current study addresses from a statistical perspective.

Univariate Time Series Models:

This section presents the linear and non-linear time series models used in the study, with an emphasis on the ARIMA and the GARCH models as standard models for modeling conditional mean and conditional variance.

The Autoregressive Moving Average Model ARMA(*p*,*q*):

The ARMA(*p*,*q*) model is a hybrid model that combines the autoregressive AR(*p*) model and the moving average MA(*q*) model, where it is widely used in time series analysis. It is expressed as follows:

$$Y_t = \varphi_0 + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \ldots - \theta_q \varepsilon_{t-q}\,, \qquad (1)$$

where $Y_t, Y_{t-1}, \ldots Y_{t-p}$ is the variable under study at time *t, t-1,……t-p, p* and *q* are the orders of the AR and the MA models, respectively, φ_0 is a constant, while $\varphi_1, \ldots, \varphi_p$ and $\theta_1, \ldots, \theta_q$ are the model coefficients and $\varepsilon_t, \ldots, \varepsilon_{t-q}$, are the error term with zero mean and positive variance σ_ε^2 with no correlations (with noise). Moreover, the ARMA(*p*,*q*) model assumes the property of stationarity, which means that the mean and variance of the data never change over time [6,24,25].

The Autoregressive Integrated Moving Average Model ARIMA(*p*,*d*,*q*):

The ARIMA(*p*,*d*,*q*) model introduced by Box and Jenkins [7], is an extension of the ARMA model by incorporating differencing of order *d* to handle non-stationarity time series. The model is expressed as follows:

$$\varphi_p(B)\,\nabla^d\,Y_t = \varphi_0 + \theta_q(B)\varepsilon_t\,, \qquad (2)$$

where $\varphi_p(B)$ and $\theta_q(B)$ represent the AR(*p*) and MA(*q*) models polynomials, respectively, $\nabla^d = (1-B)^d$ is the differencing operator, and ε_t represents a with noise error term.

The Generalized Autoregressive Conditional Heteroscedasticity model GARCH(*p*,*q*):

The GARCH(p,q) model proposed by Bollerslev [14] makes the current conditional variance (σ_t^2) depend on the squares of the prior innovations of order *p* and the prior conditional variance of order$q$. The formula for this model is given as follows:

$$w_t = \mu_t + \varepsilon_t \quad , \quad \varepsilon_t = \sigma_t z_t \quad , \quad z_t \sim iidN(0,1),$$
$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i\,\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\,\sigma_{t-j}^2\,, \qquad (3)$$

where μ_t is a conditional mean of w_t , α_0 is a constant and $\alpha_0 > 0$, *p* is the order of the ARCH term, *q* is the order of the GARCH term, $\alpha_i \geq 0$, $\beta_j \geq 0$ are model parameters and z_t is a white noise. Moreover, stationarity requires $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$.

Hybrid ARIMA-GARCH Model:

The hybrid ARIMA-GARCH model combines ARIMA for modeling the conditional mean with GARCH for capturing conditional heteroskedasticity in the residuals. In this framework, ARIMA accounts for linear dependence in the series, while GARCH deals with time-varying volatility when ARCH effects are present.

The Error Distributions of GARCH Models:

The traditional ARCH/GARCH models assume that the error limit follows a normal distribution. However, subsequent studies have proposed more suitable alternatives, such as the Student's *t*-distribution, and the generalized error distribution (GED), to represent the features of thick tails in financial series (see, e.g., [14, 26,27]).

Methodology:

Applying the hybrid ARIMA-GARCH models for study data consists of main four steps,: first, identifying the optimal ARIMA model to eliminate any linear dependence in data; second, testing the model residuals to detect potential ARCH effects, third, if ARCH effects are statistically significant, using the GARCH model and estimating the coefficients of both models together, and finally evaluate the forecasting accuracy of the models.

Model identification:

The first step in the Box-Jenkins methodology requires verifying the stationarity of the time series. Stationarity can be checked using the plots of the autocorrelation function (ACF) and the partial autocorrelation function, as well as unit root tests, such as the Augmented Dickey-Fuller (ADF) test [28]. If the stationarity of the series is not achieved, a suitable mathematical transformation or the difference of order *d* is applied to stabilize the series.

The Augmented Dickey Fuller (ADF) Test:

The ADF test is used to verify the stationarity of the time series by testing for the presence of a unit root. The null hypothesis (H_0: data has a unit root) is rejected if the *p*-value is less than the significance level α, and the ADF statistic is calculated as follows:

$$ADF\ test = \frac{\hat{\phi}_1 - 1}{SE\,(\hat{\phi}_1)}\,, \qquad (4)$$

where $\hat{\phi}_1$ is the estimator of the least squares of φ_1 with the standard error is $SE(\hat{\phi}_1)$. However, after that for identifying the model, the patterns of ACF and PACF are use to determine the optimal orders of AR(*p*) and MA(*q*). Table 1 shows the ACF and the PACF patterns of Box-Jenkins models.

Table (1): The Patterns of the ACF and the PACF.

Model	AR(*p*)	MA(*q*)	ARMA(*p*,*q*)
ACF	Dies down	Cut off after lag *q*	Dies down
PACF	Cut off after lag *p*	Dies down	Dies down

Estimation of model parameters:

After determining the appropriate model, the model parameters are usually estimated using the Maximum Likelihood Estimation (MLE) method, which is a very efficient and accurate method for estimating parameters in time series models.

Selecting the Best Model:

To select the best model from a set of candidates, information criteria such as the Akaike Information Criterion (AIC) [29] and the Schwarz Information Criterion (SIC) [30] are used. These criteria are calculated as follows:

$$AIC = -2\,ln(L) + 2k\ , \qquad (5)$$

$$SIC = 2\,ln(L) + k\,ln(T) \qquad (6)$$

where L denotes the likelihood function of the model, k is the number of estimated parameters, and T is the size of sample. The model that gives the lowest values of these criteria is preferred.

Diagnostic checking:

After estimating the model, its suitability is assessed by analyzing the model residuals. Common diagnostic tests include:

Independence tests: the ACF plots and the Ljung-Box test [31] are used to check for autocorrelation in the residuals. Moreover, the Ljung-Box test statistic is calculated as follows:

$$Q(m) = T(T+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{t-k}\ , \qquad (7)$$

where Q is the test statistic, T is the size of sample, m is the number of lags to be tested and $\hat{\rho}_k$ is an autocorrelation coefficient at lag k. If the p-value is small (< α), the null hypothesis is rejected and the residuals are found to be autocorrelated.

Normality distribution tests: the histogram and the Jarque-Bera (JB) test [32] are used to assess how closely the residuals approximate a normal distribution. However, the JB test statistic is calculated as follows:

$$JB = \frac{\hat{S}^2(y)}{6/T} + \frac{(\hat{K}(y)-3)^2}{24/T}, \qquad (8)$$

where T is the sample size and $\hat{S}^2(y)$ is skewness and $\hat{K}(y)$ is kurtosis. If the p-value is small (< α), the null hypothesis is rejected, indicating that the data are not normal.

Heteroscedasticity tests: the Ljung-Box test is used to detect significant autocorrelations for squares of residual, while the ARCH-LM test [13] is used to check for ARCH effects. If these effects are statistically significant (p-value < α), conditional variance models are used in the subsequent analysis.

Forecasting:

After verifying the suitability of the model, it used to predict future values of the series. The accuracy of the prediction is evaluated using two common measures; the root mean square error (RMSE) and the mean absolute error (MAE).

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}_t - y_t)^2}, \quad \text{and} \quad MAE = \frac{1}{T}\sum_{t=1}^{T}|\hat{y}_t - y_t|, \qquad (9)$$

where y_t represents the actual value, $\hat{y}_t$ represents the predictive value and T represents the size of sample. Therefore, the model which achieves the lowest values for these two measures is preferred.

Data Description:

This study uses monthly domestic crude oil prices for Libya (LOP) and Algeria (AOP) from January 1983 to June 2025, obtained from OPEC's Monthly Oil Market Reports [16]. The data are split into in-sample observations for model estimation and selection, and out-of-sample observations covering the final year (July 2024 to June 2025) for forecasting evaluation.

Results and discussion:

Figure 1 displays the time series of both LOP and AOP, and their correlogram from January 1983 to June, 2025. Visual inspection indicates decreasing and increasing changes at different intervals in both series, suggesting a general trend, with no evident seasonal patterns. The correlogram plots of the ACF and the PACF indices up to lag 15 exhibit slow decay for both series, indicating non-stationary.

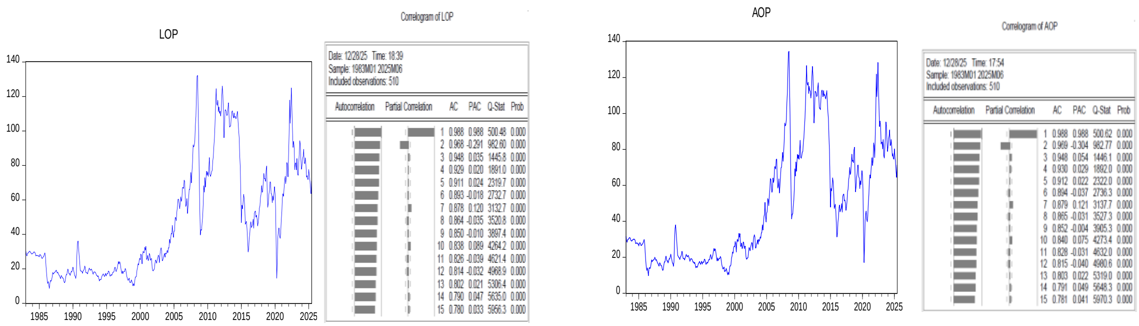

Figure (1): Plots of LOP, AOP Series and their Correlogram from January 1983 to June 2025.

Therefore, the first difference of the natural logarithm of the price series (dllop and dlaop) was computed and these new series are known as returns in the financial literature and can be defined as follows:

$$LOP = log\left(\frac{OP_t}{OP_{t-1}}\right) = log(OP_t) - log(OP_{t-1}),$$

where *LOP* is a return series, OP_t and OP_{t-1} the prices at time *t* and $t-1$. As shown in Figure 2, the differenced series fluctuated around a constant mean with stable variance, indicating stationarity. Moreover, the ACFs decay rapidly after lag 1, while and the PACFs plots cutoff after lag 2, providing further evidence that the return series are stationary.

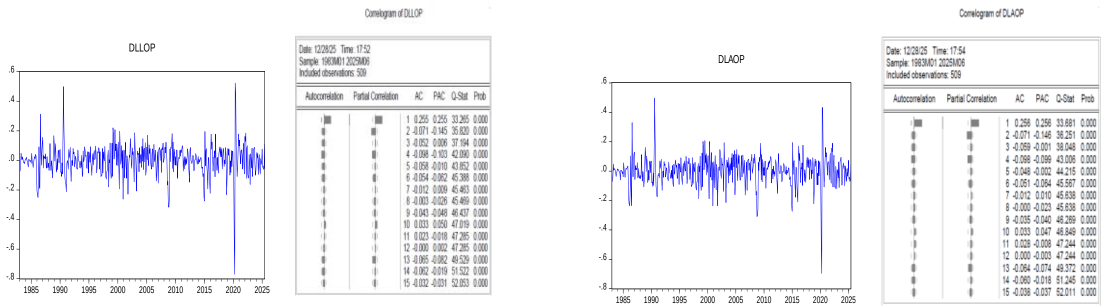

Figure (2): Plots of dllop, dlaop Series and their Correlogram from January 1983 to June 2025.

Table 2 reports the descriptive statistics of monthly prices and returns across the full sample in both countries, showing broadly similar patterns. The return series (dllop and dlaop) exhibit small positive means and relatively low standard deviations. The prices in level are positively skewed, while returns display negative skewness. Excess kurtosis is positive for both prices and returns, suggesting leptokurtic distributions. Moreover, the JB test strongly rejects the null hypothesis of normality.

Table (2): Descriptive Statistics of the Study Data.

Descriptive Statistics	Libya		Algeria	
	LOP	dllop	AOP	dlaop
Mean	47.405	0.002	48.521	0.002
Std. Dev.	32.138	0.104	32.478	0.099
Skewness	0.775	-0.795	0.752	-0.679
Kurtosis	2.406	12.956	2.359	11.098
Jarque-Bera	58.518*	2155.983*	56.766*	1429.989*
Probability	0.000	0.000	0.000	0.000
Observations	510	509	510	509

* Rejection at 1% significance level.

Unit root properties were examined using the ADF test for both prices and return series. The test was performed under specifications including an intercept and trend, intercept only, and no deterministic components, with results showed in Table 3.

Table (3): Results of Unit Root Tests.

Augmented Dickey-Fuller (ADF) Test		Libya		Algeria	
		LOP	dllop	AOP	dlaop
None	Test statistic	-1.056	-15.866*	-1.067	-15.873*
	p-value	0.263	0.000	0.259	0.000
Intercept	Test statistic	-2.867	-15.856*	-2.375	-15.864*
	p-value	0.157	0.000	0.149	0.000
Intercept and linear trend	Test statistic	-3.532	-15.843*	-3.583	-15.851*
	p-value	0.037**	0.000	0.032**	0.000
Decision		Non-stationary	Stationary	Non-stationary	Stationary

*and ** Rejection at 1% and 5% significance levels.

Table 3 shows that the *p*-values of the ADF for LOP and AOP series exceed the 1% significance level, implying that the null hypothesis is accepted and the two series are non-stationary. In contrast, the *p*-values for the differenced series (dllop and dlaop) are below 5%, confirming stationarity. To identify the mean equations different ARMA models with $p \leq 2$ and $q \leq 1$ were estimated using the MLE method. The AR and MA orders were determined by the ACF and the PACF (Figure 2), which suggest *q* = 1 and *p* = 1 or 2. Accordingly, a set of mixed ARIMA models was constructed and compared using the AIC and SIC criteria, with results showed in Table 4.

Table (4): Information Criteria Comparison of the ARIMA Models.

ARIMA (*p*,*d*,*q*)	Libya		Algeria	
	AIC	SIC	AIC	SIC
ARIMA (1,1,0)	-1.742831	-1.725895	-1.823082	-1.806146
ARIMA (0,1,1)	-1.761053	-1.744117	-1.841180	-1.824244
ARIMA (1,1,1)	-1.761863	-1.736459	-1.841530	-1.816126
ARIMA (2,1,0)	-1.759827	-1.734423	-1.840419	-1.815015
ARIMA (2,1,1)	-1.757854	-1.723982	-1.838842	-1.804970

Table 4 shows that, the AIC selects ARIMA(1,1,1) as the optimal model, while SIC suggests ARIMA(0,1,1). Parameters of both selected models were estimated, with results in Table 5. Results indicating that all coefficients in the both ARIMA(0,1,1) and ARIMA(1,1,1) models for LOP and AOP are significant at the 1% level. Following the estimation phase, several diagnostic tests were used to explore the effectiveness of the selected models, and the results are presented at the bottom of Table 5.

Table (5): Results of Estimation and Diagnostic Tests for Selected ARIMA Mosels.

Model Parameter	Libya		Algeria	
	ARIMA (0,1,1)	ARIMA (1,1,1)	ARIMA (0,1,1)	ARIMA (1,1,1)
$\varnothing_1$	-	-0.221**	-	-0.214**
θ_1	0.322*	0.523*	0.322*	0.517*
$\hat{\sigma}^2$	0.009*	0.009*	0.009*	0.009*
Diagnostic tests				
Q(20)	24.780	22.861	22.993	21.540
$Q^2(20)$	200.71*	221.19*	193.97*	208.50*
JB	1822.64*	1564.64*	985.75*	865.97*
Heteroskedasticity test: ARCH(1) and ARCH(10)				
F-statistic	224.24*	266.87*	214.86*	245.20*
nR^2	154.85*	173.97*	150.34*	164.53*
F-statistic	25.59*	30.83*	23.65*	27.12*
nR^2	170.28*	191.45*	161.64*	176.76*

*and ** Rejection at 1% and 5% significance levels.

The results of diagnostic tests on the residuals of the selected ARIMA models (Table 5) confirm that they behave as white noise (Ljung-Box test, p-values > 0.01), while the Ljung-Box test of the squared residuls suggests that the squared residuals are not independent, and there is the precense of ARCH effect (*p*-values < 0.01). The JB tests reject normality in the two models. The F-statistics and nR^2 in ARCH tests indicate that the ARCH effect is evident on the squared residuals (*p*-values < 0.01), so conditional variance must be modeled. Consequently, multiple hybrid ARIMA-GARCH models were estimated using normal, Student-t, and GED error distributions, and GARCH orders were set to $p \leq 1$ and $q \leq 1$. Initial estimation of 18 hybrid models showed that ARIMA-GARCH(1,1) models were unstable ($\alpha+\beta > 1$) and were excluded, leaving 12 candidate models. AIC and SIC values for these models summarized in Table 6, were used to select the final specification.

Table (6): Information Criteria Comparison for the ARIMA-GARCH Models.

ARIMA-GARCH/ Error Distribution	Libya		Algeria	
	AIC	SIC	AIC	SIC
ARIMA (0,1,1)-GARCH(1,0)-N	-2.075569	-2.050165	-2.117785	-2.092381
ARIMA (0,1,1)-GARCH(1,0)-t	-2.098990	-2.065118	-2.138500	-2.104628
ARIMA (0,1,1)-GARCH(1,0)-GED	-2.088252	-2.053380	-2.129354	-2.095482
ARIMA (0,1,1)-GARCH(0,1)-N	-1.804594	-1.779191	-1.872465	-1.847061
ARIMA (0,1,1)-GARCH(0,1)-t	-2.008677	-1.974805	-2.050341	-2.016469
ARIMA (0,1,1)-GARCH(0,1)-GED	-1.963160	-1.929288	-2.008166	-1.974294
ARIMA (1,1,1)-GARCH(1,0)-N	-2.072894	-2.038970	-2.120263	-2.086339
ARIMA (1,1,1)-GARCH(1,0)-t	-2.096254	-2.054849	-2.138701	-2.096296
ARIMA (1,1,1)-GARCH(1,0)-GED	-2.088294	-2.045889	-2.132771	-2.090366
ARIMA (1,1,1)-GARCH(0,1)-*N*	-1.808970	-1.775046	-1.875867	-1.841943
ARIMA (1,1,1)-GARCH(0,1)-*t*	-2.012061	-1.969656	-2.051066	-2.008661
ARIMA (1,1,1)-GARCH(0,1)-*GED*	-1.967137	-1.924732	-2.008893	-1.966488

Table 6 shows for LOP, the optimal model according to both AIC and SIC is the ARIMA(0,1,1)-GARCH (1,0)-*t* model with the Student-*t* distribution, followed by ARIMA(1,1,1)-GARCH(1,0)-*t* model. For AOP, the AIC suggests the ARIMA(1,1,1)-GARCH(1,0)-*t*, while the SIC selects the ARIMA(0,1,1)-GARCH(1,0)-*t*. Forecast performance was compared between these candidate models, and their estimated coefficients are presented in Table 7. The estimated coefficients for the two selected models (Table 7) are statistically significant at the 1% level in both mean and variance equations, except for AR(1) in ARIMA(1,1,1,)-GARCH1(1,0) for LOP, which is significant at 10%. Diagnostic tests indicate that both residuals and the squared residuals are uncorrelated (Ljung-Box test, *p*-values > 0.01). Normality is rejected; while the ARCH tests up to lag 10 confirm the absence of conditional heteroscedasticity.

Table (7): Results of Estimation and Diagnostic Tests for Selected ARIMA-GARCH Mosels.

Model Parameter	Libya		Algeria	
	ARIMA (0,1,1) GARCH(1,0)-t	ARIMA (1,1,1) GARCH(1,0)-t	ARIMA (0,1,1) GARCH(1,0)-t	ARIMA (1,1,1) GARCH(1,0)-t
Mean equation				
$\varnothing_1$	-	-0.298	-	-0.424*
θ_1	0.203*	0.490*	0.226*	0.621*
Variance equation				
α_0	0.004*	0.004*	0.004*	0.004*
α_1	0.551*	0.546*	0.496*	0.506*
T-DIST. D	9.318*	9.053*	9.907*	9.867*
Diagnostic tests				
Q(20)	21.211	22.001	17.771	20.469
$Q^2(20)$	24.991	24.774	24.096	25.029
JB	62.903*	61.023*	59.923*	50.186*
Heteroskedasticity test: ARCH(10)				
F-statistic	0.993	1.047	0.863	0.992
nR^2	9.954	10.479	8.669	9.945

* Rejection at 1% significance level.

Finally, the out-of-sample forecast accuracy for the selected ARIMA-GARCH models was evaluated using the past 12 months from July 2024 to June 2025. Forecast errors, including RMSE and MAE, are calculated and reported in Table 8.

Table (8): Out-of-Sample Forecasting Performance of the ARIMA-GARCH Models.

Out-of-sample July 2024 to June 2025		Forecast Evaluation Statistics			
		ARIMA(0,1,1)-GARCH(1,0)-t		ARIMA(1,1,1)-GARCH(1,0)-t	
series		RMSE	MAE	RMSE	MAE
Libya	LOP	4.294	3.772	4.177	3.661
	dllop	0.059	0.051	0.058	0.050
Algeria	AOP	4.152	3.707	3.979	3.543
	dlaop	0.056	0.049	0.054	0.047

The comparisons of the out-of-sample prediction accuracy indicate that the ARIMA(1,1,1)-GARCH(1,0)-*t* model performs best based on the RMSE and MAE measures, with lower values compared to the ARIMA(0,1,1)-GARCH(1,0)-*t* model. In this case, the finding clearly demonstrate that the ARIMA(1,1,1)-GARCH(1,0)-*t* model for both LOP and AOP is the best forecasting model. Figures 3 and 4 show the residuals plots and histograms for LOP and AOP using the best-fitting ARIMA(1,1,1)-GARCH(1,0)-*t* model.

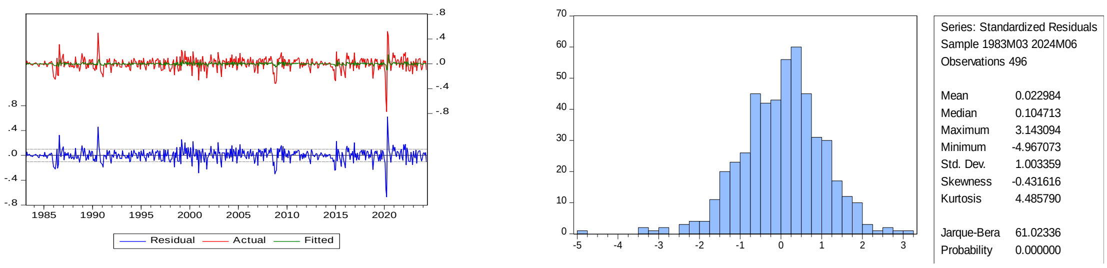

Figure (3): Plot and Histogram of Residuals for LOP using ARIMA(1,1,1)-GARCH(1,0)-t Model.

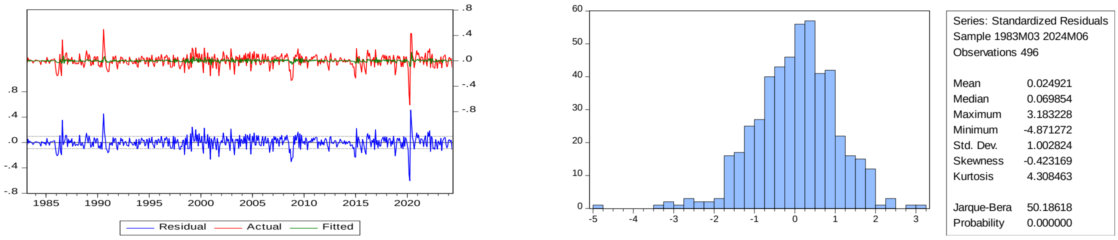

Figure (4): Plot and Histogram of Residuals for AOP using ARIMA(1,1,1)-GARCH(1,0)-t Model.

Conclusion:

This study employed hybrid ARIMA-GARCH models with normal, Student-*t*, and generalized error distributions to model monthly domestic oil prices in Libya and Algeria over the period January 1983-June 2025. Model selection based on AIC and SIC across twelve competing specifications identified the ARIMA(0,1,1)-GARCH(1,0)-*t* and ARIMA(1,1,1)-GARCH(1,0)-*t* models as the best fit. Forecast evaluation using RMSE and MAE measures indicated that the ARIMA(1,1,1)-GARCH(1,0)-*t* model provides superior predictive accuracy. These findings highlight the importance of heavy-tailed error distributions and volatility clustering in modeling domestic oil price dynamics, and demonstrating that the ARIMA-GARCH models provide reliable short-term forecasting efficiency, and contribute to supporting economic and policymakers in the two countries under study.

References:

1. Regnier, E. (2007). Oil and energy price volatility. Energy Economics, 29(3), 405-427.
2. Salisu, A. A., & Fasanya, I. O. (2012). Comparative performance of volatility models for oil price. International Journal of Energy Economics and Policy, 2(3), 167-183.
3. Sadorsky, P. (2006). Modeling and forecasting petroleum futures volatility. Energy Economics, 28(4), 467-488.
4. Hamilton, J. D. (1983). Oil and the macroeconomy since World War II. Journal of Political Economy, 91(2), 228-248.
5. Frey, G., Manera, M., Markandya, A., & Scarpa, E. (2009). Econometric models for oil price forecasting: A critical survey. CESifo Forum, 10(1), 29-44.
6. Tsay, R. S. (2005). Analysis of financial time series (2nd ed.). John wiley & sons.
7. Box, G., & Jenkins, G. M. (1976). Time Series Analysis: Forecasting and Control (Revised ed.). Holden-Day.
8. Zhang, G. P. (2003). Time series forecasting using a hybrid ARIMA and neural network model. Neurocomputing, 50, 159-175.
9. ESQ, M. D. C., & ESQ, P. K. P. (2015). How effective are quantitative methods in forecasting crude oil prices?. Journal of financial management & analysis, 28(1), 1-15.
10. Yaziz, S. R., Ahmad, M. H., Nian, L. C., & Muhammad, N. (2011). A comparative study on Box-Jenkins and Garch models in forecasting crude oil prices. Journal of Applied Sciences, 11(7), 1129-1135.
11. Xie, W., Yu, L., Xu, S., & Wang, S. (2006). A new method for crude oil price forecasting based on support vector machines. In International Conference on Computational Science (pp. 444-451). Springer.
12. MEADE, N., & Cooper, A. (2007). An Analysis of Oil Price Behaviour. Quantitative and Qualitative Analysis in Social Sciences, 1(2), 33-54.
13. Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50(4), 987-1007.
14. Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307-327.
15. Ramirez, S. C., Arreola, L. G., & Grajeda, M. R. (2012). Non-linear dependence in oil Price behavior. Journal of Mathematics and System Science, 2(2), 85-94.
16. The Organization of the Petroleum Exporting Countries. (2024). OPEC official website. https://www.opec.org/opec_web/en/17.htm
17. Wang, S., Yu, L., & Lai, K. K. (2004). A novel hybrid AI system framework for crude oil price forecasting. In CAS Symposium on Data Mining and Knowledge Management (pp. 233-242). Springer.
18. Mohammadi, H., & Su, L. (2010). International evidence on crude oil price dynamics: Applications of ARIMA-GARCH models. Energy Economics, 32(5), 1001-1008.
19. Ahmed, R. A., & Shabri, A. B. (2014). Daily crude oil price forecasting model using arima, generalized autoregressive conditional heteroscedastic and support vector machines. American Journal of Applied Sciences, 11(3), 425-432.

20. Tularam, G. A., & Saeed, T. (2016). Oil-price forecasting based on various univariate time-series models. American Journal of Operations Research, 6(3), 226-235.
21. Herrera, A. M., Hu, L., & Pastor, D. (2018). Forecasting crude oil price volatility. International Journal of Forecasting, 34(4), 622-635.
22. Shah, D. N., & Thaker, M. (2023). Forecasting Indian crude oil Price Using Hybrid ARIMA-GARCH Model. International Journal of Engineering Research and Technology (IJERT), 12(11). 1-6.
23. Perrelli, R. (2001). Introduction to ARCH & GARCH models. University of illinois optional TA handout, 1-7.
24. Maddala, G. S., & Kim, I. M. (1998). Unit roots, cointegration, and structural change. Cambridge University Press.
25. Hamilton, J. D. (1994). Time series analysis. Princeton university press.
26. Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. The Review of Economics and Statistics, 69(3),542-547.
27. Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59(2), 347-370.
28. Dickey, D. A., & Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root. Econometrica, 49(4)1057-1072.
29. Akaike, H. (1998). Information theory and an extension of the maximum likelihood principle. In Selected papers of Hirotugu Akaike (pp. 199-213). New York, NY: Springer New York.
30. Schwarz, G. (1978). Estimating the dimension of a model. The Annals of Statistics. 6(2), 461-464.
31. Ljung, G. M., & Box, G. E. (1978). On a measure of lack of fit in time series models. Biometrika, 65(2), 297-303.
32. Jarque, C. M., & Bera, A. K. (1987). A test for normality of observations and regression residuals. International Statistical Review, 55(2)163-172.