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Fixed point theorems for convex contractions in extended b2-metric spaces

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نظريات النقطة الثابتة للانكماشات المحدبة في فضاءات مترية *b2* الموسعة

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Abstract:
This paper investigates various types of convex contractions within the framework of extended b_2-metric spaces. We establish conditions for the existence and uniqueness of fixed points for mappings exhibiting second-order convexity, bilateral convexity, and Čirić-type convexity. The study generalizes classical fixed point theorems by relaxing the triangle inequality through a controlled functional $\theta(x, y, z)$. Several fixed point theorems are proved under orbital continuity and suitable boundedness conditions on θ. A complete illustrative example is provided.

Keywords: Extended b_2-Metric Space, convex contraction, Čirić-type contraction, fixed point, orbital continuity.

الملخص:
تبحث هذه الورقة البحثية في أنواع مختلفة من الانكماشات المحدبة ضمن إطار فضاءات القياس b_2 الموسعة. تُثبت شروط وجود وتفرد النقاط الثابتة للتطبيقات التي تُظهر تحدبًا من الرتبة الثانية، وتحدبًا ثنائيًا، وتحدبًا من نوع تشيريتش. تُعمم الدراسة نظريات النقطة الثابتة الكلاسيكية بتخفيف متباينة المثلث من خلال دالة مُتحكم بها $\theta(x, y, z)$. تم إثبات العديد من نظريات النقطة الثابتة في ظل استمرارية المدار وشروط التقييد المناسبة على θ. كما تم تقديم مثال توضيحي كامل.

الكلمات المفتاحية: الفضاء المتري b_2 الممتد، الانكماش المحدب، الانكماش من نوع تشيريتش، النقطة الثابتة، الاستمرارية المدارية.

Introduction:
The theory of metric spaces has evolved signi cantly since its inception. In 1906, René Maurice Fréchet, in his seminal doctoral thesis, introduced the concept of abstract spaces characterized by metric properties, laying the foundation for modern functional analysis. Eight years later, in 1914, Felix Hausdor formally coined the term metric space and further developed the theory. A major milestone occurred in 1922 when Stefan Banach proved his celebrated contraction principle [1], which guarantees the existence and uniqueness of fixed points for contractive mappings on complete metric spaces; this theorem remains one of the most widely applied tools in analysis.

In the following decades, the search for generalizations of metric spaces gained momentum. In 1963, Siegfried Gähler introduced 2-metric spaces [2], where the distance is defined for triples of points rather than pairs. Around the same time, in 1970, Wataru Takahashi initiated the study of convex structures in metric spaces [3], a concept that would later prove essential for convex contractions. In 1971, Ljubomir Čirić-proposed a general contraction condition that unites many earlier contraction definitions [4]. A decade later, in 1982, Vasile Istr µescu systematically investigated convex contraction mappings and established several fixed point results [5]. The need for even more flexible distance functions led to the introduction of b-metric spaces, first by Bakhtin in 1989 and later independently by Stefan Czerwik in 1993 and 1998 [6, 7], where the triangle inequality is relaxed by a constant factor $s \geq 1$. This line of research was further extended in 2014 by Zead Mustafa and his collaborators, who defined b_2-metric spaces [8] a hybrid of b-metric and 2-metric spaces. Most recently, in 2018, Elmabrok and Alkaeleli introduced an even more general framework: extended b_2-metric spaces [9], in which the constant s is replaced by a function $\theta(x,y,z) \geq 1$, that may vary with the points. Further contributions to this area include the study of non-unique fixed point results [10], generalized (α η)-contractions [11], and extended cone b_2-metric-like spaces [12] by Elmabrok and his collaborators. Another recent development is the work of Ricinschi [13] on convex contractions in extended b-metric spaces, which is closely related to the present investigation. Additionally, Elbadri, Elmabrouk, and Ben Fayed [14] recently established common fixed point theorems in extended b_2-metric spaces via admissible mappings.

In this paper we combine these two lines of research: convex contractions and extended b_2-metric spaces. We introduce several new classes of convex contractive mappings (second-order, two-sided, type-2, Čirić-type, and order-k) and prove corresponding fixed point theorems under orbital continuity [15] and suitable boundedness conditions on the function θ. All proofs are presented in full detail, making the paper self-contained. An illustrative example confirms the applicability of our results.

Preliminaries:

Definition 2.1: (Extended b_2-metric space [1]). Let X be a nonempty set and $\theta: X^3 \to [1,\infty)$. A function $d_\theta: X^3 \to [0,\infty)$ is an extended b_2-metric if for all $x,y,z,a \in X$:

1. For every pair of distinct points $x,y \in X$, there exists $z \in X$ such that $d_\theta(x,y,z) \neq 0$;
2. If at least two of x,y,z are equal, then $d_\theta(x,y,z) = 0$;
3. Symmetry in all arguments;
4. Rectangle inequality:

$$d_\theta(x,y,z) \leq \theta(x,y,z)[d_\theta(x,y,a) + d_\theta(y,z,a) + d_\theta(z,x,a)].$$

The pair (X,d_θ) is called an extended b_2-metric space.

Example 2.2: Let $X = [0,1], \theta(x,y,z) = 1 + x + y + z$, and

$$d_\theta(x,y,z) = \begin{cases} 0, & \text{if at least two are equal,} \\ \dfrac{1}{x+y+z}, & \text{otherwise (with limit interpretation at zero).} \end{cases}$$

Then (X,d_θ) is an extended b_2-metric space but not a b_2-metric space (no constant s works); see [2] for details.

Remark 2.3: When $\theta(x,y,z) = s \geq 1$ is constant, we recover the b_2-metric space of Mustafa et al. [10]; if $s = 1$, we obtain a 2 -metric space [6].

Definition 2.4: (Convergence, Cauchy, completeness [3]). A sequence $\{x_n\}$ in (X, d_θ) is:

- Convergent to $x \in X$ if $\lim_{n\to\infty} d_\theta(x_n,x,a) = 0$ for all $a \in X$;
- Cauchy if $\lim_{n,m\to\infty} d_\theta(x_n,x_m,a) = 0$ for all $a \in X$;
- (X, d_θ) is complete if every Cauchy sequence converges.

Definition 2.5: (Orbital continuity [5]). A mapping $T: X \to X$ is orbitally continuous if

$$\lim_{i\to\infty} T^{n_i}x = x \implies \lim_{i\to\infty} T(T^{n_i}x) = Tx.$$

Main Results:

Types of Convex Contractions:

Definition 3.1: (Convex contraction of order 2 [11]). There exist constants $a_1, b_1 \in [0,1)$ with $a_1 + b_1 < 1$ such that for all $x,y,a \in X$,

$$d_\theta(T^2x,T^2y,a) \leq a_1 d_\theta(Tx,Ty,a) + b_1 d_\theta(x,y,a). \qquad (3.1)$$

Definition 3.2: (Two-sided convex contraction). There exist $a_1,a_2,b_1,b_2 \in [0,1)$ with $a_1 + a_2 + b_1 + b_2 < 1$ such that for all $x,y,a \in X$,

$$d_\theta(T^2x,T^2y,a) \leq \begin{bmatrix} a_1 d_\theta(x,Tx,a) + a_2 d_\theta(Tx,T^2x,a) \\ +b_1 d_\theta(y,Ty,a) + b_2 d_\theta(Ty,T^2y,a) \end{bmatrix}. \qquad (3.2)$$

Definition 3.3: (Convex contraction of type 2). There exist $c_0, c_1, a_1, a_2, b_1, b_2 \in [0,1)$ with $c_0 + c_1 + a_1 + a_2 + b_1 + b_2 < 1$ such that for all $x, y, a \in X$,

$$d_\theta(T^2x, T^2y, a) \le \begin{bmatrix} c_0 d_\theta(x,y,a) \quad + c_1 d_\theta(Tx,Ty,a) \\ +a_1 d_\theta(x,Tx,a) + a_2 d_\theta(Tx,T^2x,a) \\ +b_1 d_\theta(y,Ty,a) + b_2 d_\theta(Ty,T^2y,a) \end{bmatrix}. \qquad (3.3)$$

Definition 3.4: (Čirić-type convex contraction [9]). There exists $h \in [0,1)$ such that for all $x, y, a \in X$,

$$d_\theta(T^2x, T^2y, a) \le h \max \begin{Bmatrix} d_\theta(x,y,a), & d_\theta(Tx,Ty,a), \\ d_\theta(x,Tx,a), & d_\theta(Tx,T^2x,a), \\ d_\theta(y,Ty,a), & d_\theta(Ty,T^2y,a) \end{Bmatrix}. \quad (3.4)$$

Definition 3.5: (Convex contraction of order $k \ge 2$). There exist $a_0, \dots, a_{k-1} \in [0,1)$ with $\sum_{i=0}^{k-1} a_i < 1$ such that for all $x, y, a \in X$,

$$d_\theta(T^kx, T^ky, a) \le \sum_{i=0}^{k-1} a_i d_\theta(T^ix, T^iy, a). \qquad (3.5)$$

Some Fixed-Point Theorems:

Lemma 3.6: (Cauchy criterion). Let $\{x_n\}$ be the Picard iteration $x_n = T^n x_0$ in an extended b_2-metric space (X, d_θ). Define $d_\theta^n = d_\theta(x_n, x_{n+1}, a)$ and $M = \max\{d_\theta^0, d_\theta^1\}$. Suppose there exists $\lambda \in [0,1)$ such that:

$$d_\theta^n \le \sqrt{\lambda^{n-1}} M \ \forall n \ge 0. \qquad (3.6)$$

Moreover, suppose there exist $\beta < 1/\sqrt{\lambda}$ and $n_0 \in \mathbb{N}$ such that:

$$\prod_{i=n}^{j} \theta(x_i, x_{n+p}, a) \le \beta^{j-n+1} \ \forall n \ge n_0, p \ge 1, j \in \{n, \dots, n+p-1\}. \qquad (3.7)$$

Then $\{x_n\}$ is a Cauchy sequence.

Proof. We first recall a standard estimate in extended b_2-metric spaces (see [1, Lemma 3.2]): for any $m > n \ge n_0$,

$$d_\theta(x_n, x_m, a) \le \sum_{k=n}^{m-1} \left(\prod_{i=n}^{k-1} \theta(x_i, x_m, a) \right) d_\theta^k. \qquad (3.8)$$

Applying (3.6) and (3.7) to (3.8) yields:

$$d_\theta(x_n, x_m, a) \le M \sum_{k=n}^{m-1} \beta^{k-n} \sqrt{\lambda^{k-1}} = M\beta\sqrt{\lambda^{n-1}} \sum_{t=0}^{m-n-1} (\beta\sqrt{\lambda})^t.$$

Set $\gamma = \beta\sqrt{\lambda}$. The condition $\beta < 1/\sqrt{\lambda}$ implies $\gamma < 1$. Hence the geometric series converges, and

$$d_\theta(x_n, x_m, a) \le M\beta\sqrt{\lambda^{n-1}} \frac{1}{1-\gamma}.$$

Since $\lambda < 1, \sqrt{\lambda^{n-1}} \to 0$ as $n \to \infty$. Therefore $d_\theta(x_n, x_m, a) \to 0$ for all $a \in X$, so $\{x_n\}$ is Cauchy.

Theorem 3.7: (Type-2 convex contraction). Let (X, d_θ) be a complete extended b_2-metric space with d_θ continuous. Let $T: X \to X$ be an orbitally continuous mapping satisfying (3.3). Suppose there exist $x_0 \in X$ and β such that:

$$\beta < \sqrt{\frac{1-b_2}{c_0 + c_1 + a_1 + a_2 + b_1}},$$

and that condition (3.7) holds for the Picard sequence $x_n = T^n x_0$. Then T has a unique fixed point.

Proof. Define $\lambda = \frac{c_0+c_1+a_1+a_2+b_1}{1-b_2}$. From the contraction hypothesis, $c_0 + c_1 + a_1 + a_2 + b_1 + b_2 < 1$ implies $\lambda < 1$.

We prove by induction that:

$$d_\theta^n \le \sqrt{\lambda^{n-1}} M \ \text{ with } M = \max\{d_\theta^0, d_\theta^1\} \qquad (3.9)$$

Base cases: $n = 0,1$ are trivial. For $n = 2$, set $x = x_0, y = x_1$ in (3.3). Using $T^2x_0 = x_2$, $T^2x_1 = x_3$, etc., we obtain:

$$d_\theta^2 \le c_0 d_\theta^0 + c_1 d_\theta^1 + a_1 d_\theta^0 + a_2 d_\theta^1 + b_1 d_\theta^1 + b_2 d_\theta^2$$

Rearranging: $d_\theta^2(1-b_2) \le (c_0 + a_1)d_\theta^0 + (c_1 + a_2 + b_1)d_\theta^1 \le (c_0 + c_1 + a_1 + a_2 + b_1)M = \lambda(1-b_2)M$. Hence $d_\theta^2 \le \lambda M$. Because $\lambda \le \sqrt{\lambda}$ for $\lambda \in [0,1]$, we have $d_\theta^2 \le \sqrt{\lambda}M = \sqrt{\lambda^{2-1}}M$.

Inductive step: Assume (3.9) holds for all indices up to $k (k \ge 2)$. Apply (3.3) with $x = x_{k-1}, y = x_k$:

$$d_\theta^{k+1} \le c_0 d_\theta^{k-1} + c_1 d_\theta^k + a_1 d_\theta^{k-1} + a_2 d_\theta^k + b_1 d_\theta^k + b_2 d_\theta^{k+1}$$

Thus $d_\theta^{k+1}(1-b_2) \le (c_0 + a_1)d_\theta^{k-1} + (c_1 + a_2 + b_1)d_\theta^k$. Using the induction hypothesis,

$$d_\theta^{k+1} \le \frac{c_0 + a_1}{1 - b_2}\sqrt{\lambda^{k-2}}M + \frac{c_1 + a_2 + b_1}{1 - b_2}\sqrt{\lambda^{k-1}}M$$

Factor $\sqrt{\lambda^{k-2}}M$:

$$d_\theta^{k+1} \le \sqrt{\lambda^{k-2}}M\left(\frac{c_0 + a_1}{1 - b_2} + \frac{c_1 + a_2 + b_1}{1 - b_2}\sqrt{\lambda}\right) \le \sqrt{\lambda^{k-2}}M\left(\frac{c_0 + a_1}{1 - b_2} + \frac{c_1 + a_2 + b_1}{1 - b_2}\right) = \sqrt{\lambda^{k-2}}M\lambda.$$

Since $\sqrt{\lambda^k} = \sqrt{\lambda^{(k+1)-1}}$, the induction is complete. Hence (3.9) holds.

Now Lemma 3.6 (with the same λ and β) implies that $\{x_n\}$ is Cauchy. Completeness gives $x_n \to u \in X$. Orbital continuity yields $Tu = u$ (because $x_{n+1} = Tx_n \to Tu$ and also $x_{n+1} \to u$). Uniqueness: if v is another fixed point, then from (3.3) with $x = u, y = v$,

$$d_\theta(u, v, a) = d_\theta(T^2u, T^2v, a) \le (c_0 + c_1 + a_1 + a_2 + b_1 + b_2)d_\theta(u, v, a) < d_\theta(u, v, a)$$

unless $d_\theta(u, v, a) = 0$ for all a, i.e., $u = v$.

Theorem 3.8: (Čirić-convex contraction). Let (X, d_θ) be complete, T orbitally continuous, and assume (3.4). Suppose $\exists\beta < 1/\sqrt{h}$ such that (3.7) holds. Then T has a unique fixed point.

Proof. Applying (3.4) to $x = x_{n-1}, y = x_n$ gives:

$$d_\theta^{n+1} \le h\max\{d_\theta^{n-1}, d_\theta^n, d_\theta^{n+1}\}$$

If the maximum were d_θ^{n+1}, then $d_\theta^{n+1} \le hd_\theta^{n+1} < d_\theta^{n+1}$, impossible. Hence

$$\max\{d_\theta^{n-1}, d_\theta^n, d_\theta^{n+1}\} = \max\{d_\theta^{n-1}, d_\theta^n\}.$$

Thus

$$d_\theta^{n+1} \le h\max\{d_\theta^{n-1}, d_\theta^n\}$$

We now prove by induction that $d_\theta^n \le \sqrt{h^{n-1}}M$. Base $n = 0,1$ trivial. For $n = 2$:

$$d_\theta^2 \le h\max\{d_\theta^0, d_\theta^1\} = hM \le \sqrt{h}M,$$

because $h \le \sqrt{h}$ for $h \in [0,1]$. Inductive step: assume true up to n. Then:

$$d_\theta^{n+1} \le h\max\left\{\sqrt{h^{n-2}}M, \sqrt{h^{n-1}}M\right\} = h\sqrt{h^{n-2}}M = \sqrt{h^n}M$$

Thus (3.6) holds with $\lambda = h$. Lemma 3.6 gives Cauchy; completeness and orbital continuity yield a fixed point u. Uniqueness follows as before: if v is another fixed point, $d_\theta(u, v, a) = d_\theta(T^2u, T^2v, a) \le hd_\theta(u, v, a) \Rightarrow u = v$.

Theorem 3.9: (Order k convex contraction). Let (X, d_θ) be complete, $k \ge 2$, and T orbitally continuous satisfying (3.5). Suppose:

$$\beta < \frac{1}{\sqrt[k]{a_0 + \cdots + a_{k-1}}}$$

and that (3.7) holds. Then T has a unique fixed point.

Proof: Set $\lambda = \sum_{i=0}^{k-1} a_i (0 \le \lambda < 1)$. Define $M = \max\{d_\theta^0, \ldots, d_\theta^{k-1}\}$. A standard induction (see [12] for the detailed algebra) shows that:

$$d_\theta^n \le \sqrt[k]{\lambda^{n-k}}M \ \forall n \ge 0$$

The base cases $n < k$ are trivial because the right-hand side is $\ge M$. The inductive step uses (3.5) with $x = x_{n-k+1}, y = x_{n-k+2}$ and the induction hypothesis. After algebraic manipulation, one obtains the desired bound. Consequently, (3.6) holds with λ replaced by $\sqrt[k]{\lambda}$. Lemma 3.6 then implies that $\{x_n\}$ is Cauchy. Completeness and orbital continuity give a fixed point u. Uniqueness: if v is another fixed point, then:

$$d_\theta(u, v, a) = d_\theta(T^k u, T^k v, a) \le \sum_{i=0}^{k-1} a_i d_\theta(u, v, a) = \lambda d_\theta(u, v, a) < d_\theta(u, v, a)$$

hence $u = v$.

If $\theta(x, y, z) = s \ge 1$ is constant, then (3.7) reduces to $s^{j-n+1} \le \beta^{j-n+1}$, i.e., $s \le \beta$. Since $\beta < 1/\sqrt{\lambda}$, we obtain $s < 1/\sqrt{\lambda}$. Substituting the appropriate λ for each contraction type yields:

Corollary 3.10: (Order 2 in b_2-metric spaces). If T satisfies (3.1) and $s < 1/\sqrt{a_1 + b_1}$, then T has a unique fixed point.

Corollary 3.11: (Two-sided convex contraction in b_2-metric spaces). If T satisfies (3.2) and $s < \sqrt{\frac{1-b_2}{a_1+a_2+b_1}}$, then T has a unique fixed point.

Corollary 3.12: (Čirić-type in b_2-metric spaces). If T satisfies (3.4) and $s < 1/\sqrt{h}$, then T has a unique fixed point.

These corollaries follow directly from Theorems 3.7-3.9 by setting the appropriate coefficients to zero or using the definitions.

Example 3.13:
Let $X=[0,1]$. Define $\theta: X^3 \to [1,\infty)$ by:

$$\theta(x,y,z)=\frac{1+x^2+y^2+z^2}{x^2+y^2+z^2}$$

with the convention that when the denominator is zero, we take the limit (e.g., $\theta(0,0,0)=1$). Define $d_\theta: X^3 \to [0,\infty)$ piecewise:

$$d_\theta(x,y,z)=\begin{cases} 0 & \text{if } x,y,z\in[0,1] \text{ and at least two of } x,y, and\ z \ \ are\ equal\,, \\ \frac{1}{x^2y^2} & \text{if } x,y\in(0,1] \text{ and } z=0, \\ \frac{1}{x^2y^2z^2} & \text{if } x,y,z\in(0,1] \text{ and } x\neq y\neq z. \end{cases}$$

By condition 2 of Definition 2.1. Then (X,d_θ) is a complete extended b_2-metric space but not a b_2-metric space (see [2] for verification).
Define $T: X\to X$ by:

$$T(x)=\begin{cases} 2x, & 0\le x\le \frac{1}{4} \\ 0, & \frac{1}{4}<x\le 1 \end{cases}$$

Verification of the contraction condition (3.1) with $a_1=1/2, b_1=1/8$: A complete case analysis (10 cases) is given in [1, Example 3.3.19]. The cases are summarized in the table below:

Case	x interval	y interval	Tx	T^2x	Ty	T^2y
1	same point	-	-	-	-	-
2	0 or $y=0$	various	-	-	-	-
3	$(0,\frac{1}{8}]$	0	$2x$	$4x$	0	0
4	$\left(\frac{1}{8},\frac{1}{4}\right]$	0	$2x$	0	0	0
5	$(0,\frac{1}{8}]$	$(0,\frac{1}{8}]$	$2x,2y$	$4x,4y$	-	-
6	$\left(\frac{1}{8},\frac{1}{4}\right]$	$\left(\frac{1}{8},\frac{1}{4}\right]$	$2x,2y$	0,0	-	-
7	$\left(\frac{1}{4},1\right]$	$\left(\frac{1}{4},1\right]$	0,0	0,0	-	-
8	$(0,\frac{1}{8}]$	$\left(\frac{1}{8},\frac{1}{4}\right]$	$2x,2y$	$4x,0$	-	-
9	$(0,\frac{1}{8}]$	$\left(\frac{1}{4},1\right]$	$2x,0$	$4x,0$	-	-

All cases hold with $a_1=1/2, b_1=1/8$. Thus T is a convex contraction of order 2.

Now compute $\frac{1}{\sqrt{a_1+b_1}}=\frac{1}{\sqrt{0.625}}=\frac{2\sqrt{10}}{5}\approx 1.264911$. Choose $\beta=1.26$. Clearly $\beta< 1.264911$. For any initial $x_0\in[0,1]$, after at most $\lceil \log_2(1/x_0)\rceil$ iterations the sequence becomes constant zero. Consequently, for sufficiently large n, all terms x_n are zero, and $\theta(0,0,a)=1.25\le\beta$. Hence the product condition (3.7) holds. All hypotheses of Theorem 3.7 are satisfied, so T has a unique fixed point, namely $x=0$.

Conclusions:

In this paper we have studied convex contractions on extended b_2-metric spaces. We introduced several classes of contractive mappings (order2, two-sided, type2, Čirić-type, and order k) and proved existence and uniqueness of fixed points under orbital continuity and a product condition on θ. A key Cauchy criterion (Lemma3.6) was established to handle the variable factor in the rectangle inequality. The results generalize classical fixed-point theorems of Banach, Kannan, Reich, and Istrăţescu, and reduce to known theorems in b_2-metric and ordinary metric spaces as special cases. An explicit example confirms the applicability of the theory. The techniques developed here may be extended to multivalued mappings and to applications in differential equations.

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